Section 1.1: THE DISTANCE AND MIDPOINT FORMULAS; GRAPHING UTILITIES; INTRODUCTION TO GRAPHING EQUATIONS

When you are done with your homework you should be able to…

- π Use the Distance Formula
- π Use the Midpoint Formula
- π Graph Equations by Hand by Plotting Points
- π Graph Equations Using a Graphing Utility
- π Use a Graphing Utility to Create Tables
- π Find Intercepts from a Graph
- π Use a Graphing Utility to Approximate Intercepts

WARM-UP:

What grade do you want to earn in this class?

For each unit, how many hours should you spend on the class?

How many hours for "class time"?

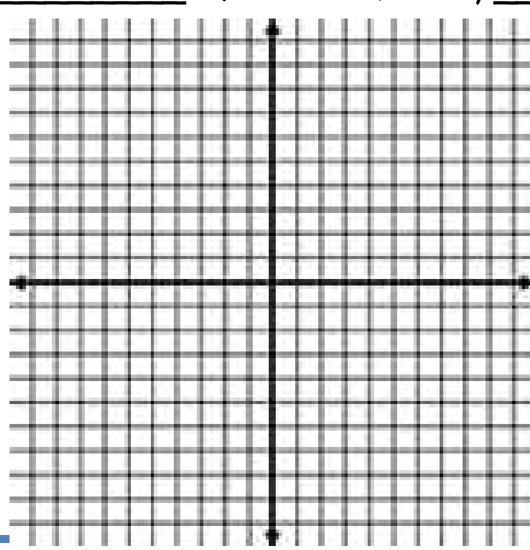
How many hours for homework, test prep, etc.?

When should you work on math?

RECTANGULAR COORDINATES

We locate a ______________ on the real number line by assigning it a single real number, called the ___________________ of the point. For work in a two-dimensional ______________, points are located by using _________ numbers. The rectangular or Cartesian coordinate system consists of _____ real number lines, one _______________ and one _________________. The horizontal line is called the _______________ and the vertical line is called the ______________. The point of intersection is located at the ordered pair ____________ and is called the ___________________. Assign ____________________ to every point on these number lines using a convenient scale. The scale of a number line is the distance between ______ and ______. Once you set the scale, it stays the same on that particular axis. Sometimes the scale on the *x*- and *y*-axes differ. For example, if you are sketching a line that has *x*-coordinates that can be easily viewed using a scale between -6 and 6 and *y*-coordinates that are better viewed between -1 and 1, you may want to set the scale for the *x*-axis as _____ and the *y*-axis as _____.

Points on the *x*-axis to the right of *O* are associated with _____________ real numbers, and those to the _________________ of *O* are associated with ___________________ real numbers. Points on the *y*-axis above *O* are associated with _____________ real numbers, and those _______________ *O* are associated with ___________________ real numbers. The ________ divide the ___________ into ___________ regions, called _______________. The points located on the _______________ are __________ in any quadrant. Each _______________ in the rectangular coordinate system _________________ to an _______________ ___________ of real numbers, __________.



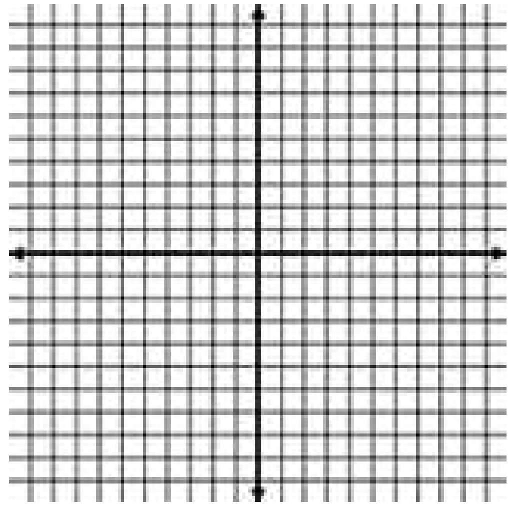
Example 1:

a. Plot the following ordered pairs. Identify which quadrant or on what coordinate axis each point lies.

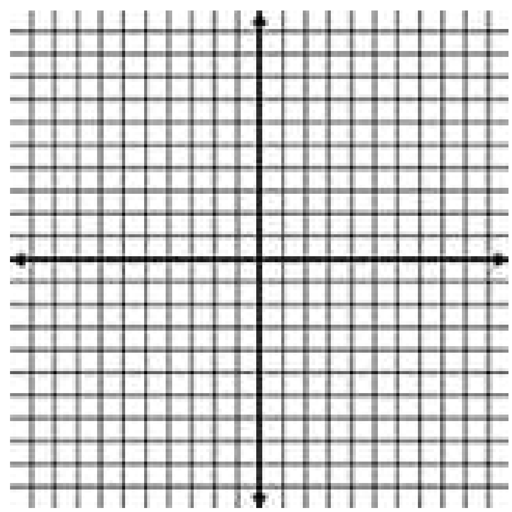
$A = (2,5)$

$B = (-3,7)$

$C = (-2,0)$



b. Plot the points $(0,3)$, $(1,3)$, $(5,3)$, $(-4,3)$. Describe the set of all points of the form $(x,3)$ where x is a real number.



GRAPHING UTILITIES

All graphing utilities graph equations by ________________ points. The screen itself consists of small rectangles, called __________________. The more pixels the screen has, the better the resolution. When a point to be plotted lies inside a pixel, the pixel is turned on (lights up). The graph of an equation is a collection of

_______________ pixels.

The screen of a graphing calculator will display the coordinate axes of a rectangular coordinate system, but you need to set the ______________ on each axis. You must also include the _______________ and ________________ values of ______ and ______ that you want included in the graph. This is called _________________ the _______________ _____________________ or __________________ __________________.

Xmin: the _______________ value of ___ shown on the viewing window

Xmax: the _______________ value of ___ shown on the viewing window

Xscl: the number of ___________ per ___________ mark on the _____________

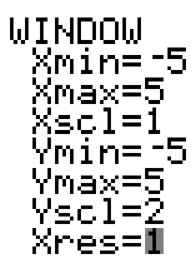
Ymin: the _______________ value of ___ shown on the viewing window

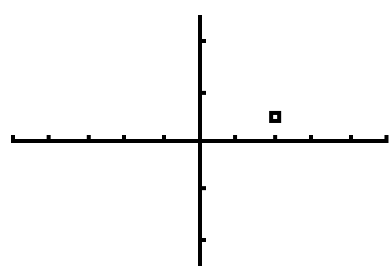
Ymax: the _______________ value of ___ shown on the viewing window

Yscl: the number of ___________ per ___________ mark on the _____________

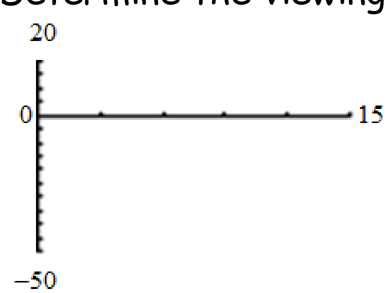
Example 2:

a. Find the coordinates of the point shown below. Assume the coordinates are integers.





b. Determine the viewing window used.



Xmin: ______

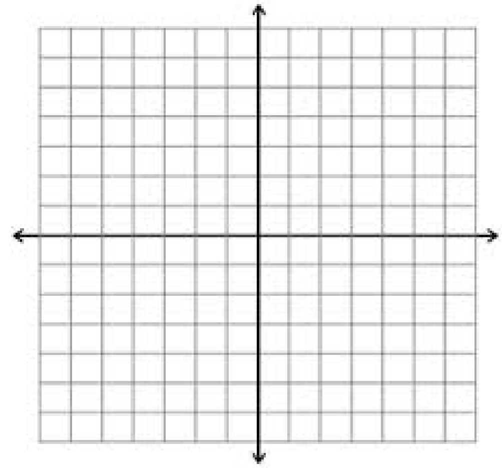
Xmax: ______

Xscl: _______

Ymin: ______

Ymax: ______

Yscl: _______



DISTANCE FORMULA

The distance between two points _______________ and ______________, denoted by _________________, is

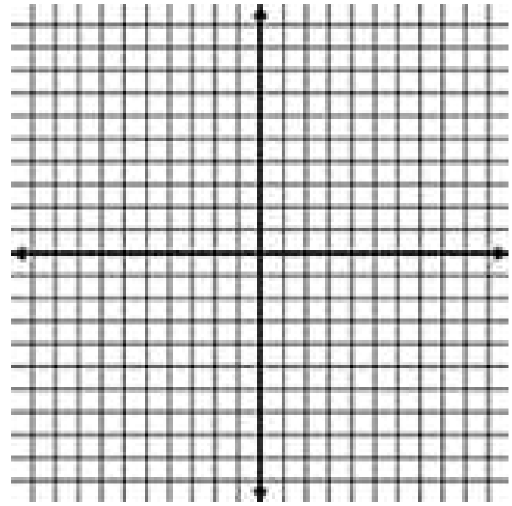
Example 3: Find the distance between each pair of points.

a. $P_1 = (-4, -3)$ and $P_2 = (6, 2)$

b. $P_1 = (a, a)$ and $P_2 = (0, 0)$

Example 4: Consider the points $A=(-2,5)$, $B=(12,3)$, and $C=(10,11)$.

a. Plot each point and form the triangle *ABC*.



b. Verify that the triangle is a right triangle.

c. Find its area

THE MIDPOINT FORMULA

Consider a line segment whose endpoints are ____________ and ____________.

The midpoint, ____________, is

Example 5: Find the midpoint of the line joining the points P_1 and P_2.

a. $P_1 = (1,4)$ and $P_2 = (-2,7)$

b. $P_1 = (a,a)$ and $P_2 = (0,0)$

GRAPH EQUATIONS BY HAND BY PLOTTING POINTS

An ________________ in _______ __________________, say ____ and ____, is a statement in which two expressions involving *x* and *y* are ________________. The expressions are called the ____________ of the equation. Since an equation is a statement, it may be _________ or _____________, depending on the value of the variables. Any values of *x* and *y* that result in a true statement are said to ____________________ the equation.

The _______________ of an _________________ in _______ ______________ *x* and *y* consists of the __________ of points in the _______ plane whose coordinates _____________ satisfy the equation.

Example 6: Tell whether the given points are on the graph of the equation.

Equation: $y = x^3 - 2\sqrt{x}$

Points: $(0,0)$; $(1,1)$; $(1,-1)$

GRAPHING EQUATIONS USING A GRAPHING UTILITY

To graph an equation in two variables *x* and *y* using a graphing calculator requires that the dependent variable, *y*, be isolated.

PROCEDURES THAT RESULT IN EQUIVALENT EQUATIONS

1. Interchange the two sides of the equation:

   ____________________ is equivalent to ______________________

2. Simplify the sides of the equation by combining like terms, eliminating parentheses, etc.:

   ____________________ is equivalent to ______________________

3. Add or subtract the same expression on both sides of the equation:

   ____________________ is equivalent to ______________________

4. Multiply or divide both sides of the equation by the same nonzero expression:

   ____________________ is equivalent to ______________________

Example 7: Solve for *y*.

a. $5-(x-3)=2y+6\left(\frac{1}{2}y-1\right)$

b. $4y-x^2=3$

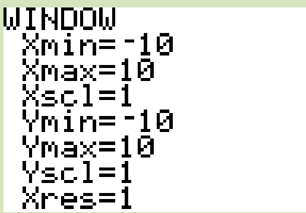
HOW TO GRAPH AN EQUATION USING THE TI-83/TI-84 GRAPHING CALCULATOR

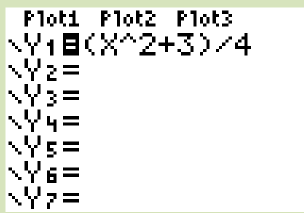
1. Solve the equation for _____ in terms of _____.

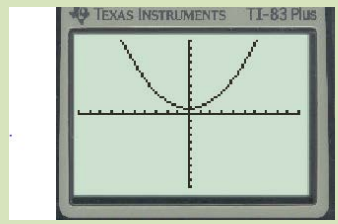
2. Enter the equation to be graphed into your graphing calculator.

   a. Use the "y =" key.

   b. Graph the equation using the standard viewing window.

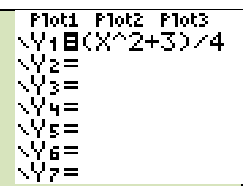






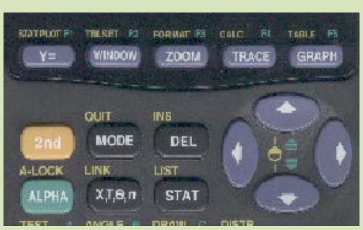
   c. Adjust the viewing window.

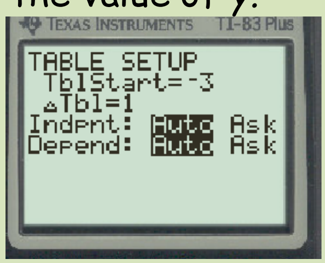
HOW TO USE THE TI-83/TI-84 TO CREATE TABLES



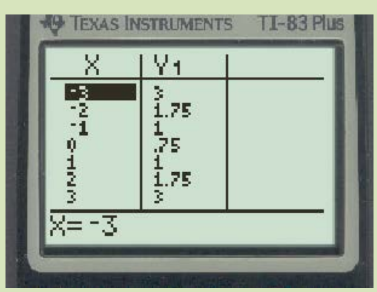
1. Solve the equation for _____ in terms of _____.
2. Enter the equation to be graphed into your graphing calculator.

3. Set up the table. In the AUTO mode, the user determines the starting point for the table and delta table, which determines the increment for *x* in the table. The ASK mode requires the user to enter values of *x*, and then the calculator determines the value of *y*.



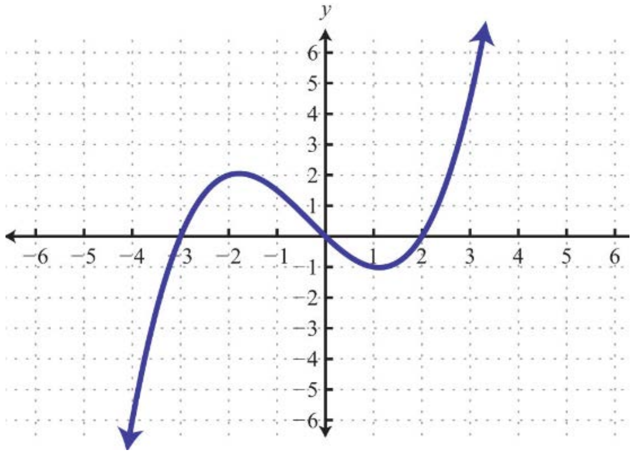


4. Create the table.

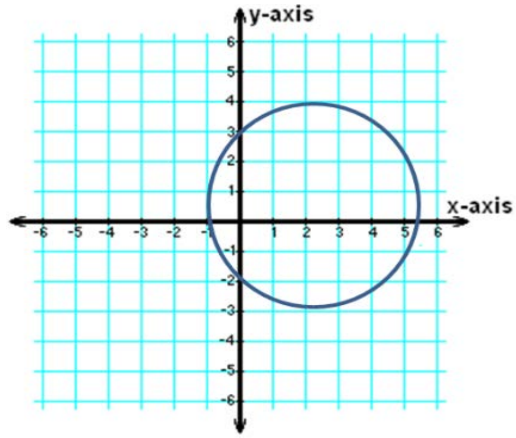


Example 8: The graph of an equation is given. List the intercepts of the graph.

a.



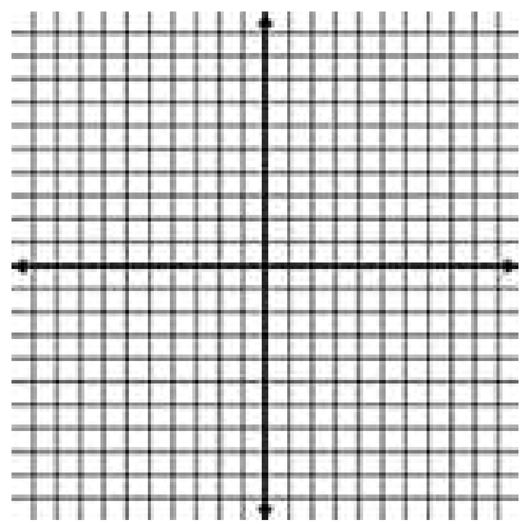
b.



Example 9: Graph each equation by hand by plotting points. Verify your results using a graphing utility.

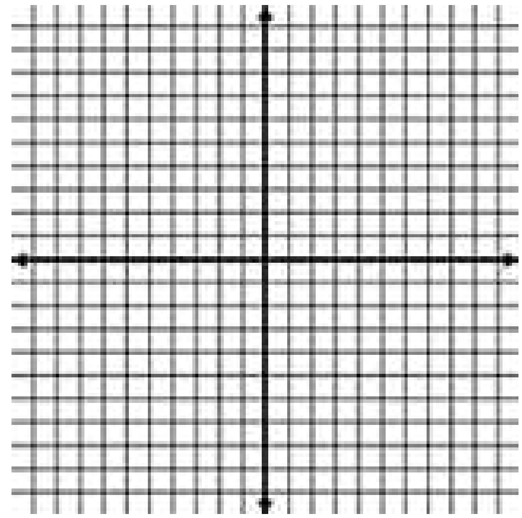
a. $y = 3x - 9$

x	$y = 3x - 9$	(x, y)



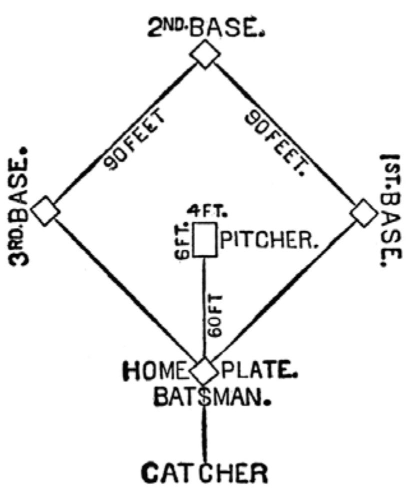
b. $y=-x^2+1$

x	$y=-x^2+1$	(x,y)



APPLICATIONS

A major league baseball "diamond" is actually a square, 90 feet on a side. What is the distance directly from home plate to second base (the diagonal of a square)? Give the exact simplified result first, and then round to the nearest hundredth.



Section 1.2: INTERCEPTS; SYMMETRY, GRAPHING KEY EQUATIONS

When you are done with your homework you should be able to…

- π Find Intercepts Algebraically from an Equation
- π Test an Equation for Symmetry
- π Know How to Graph Key Equations

Warm-up: Solve.

a. $3x-4(2x-8)=3-5x$

b. $2x^2-x=3$

PROCEDURE FOR FINDING INTERCEPTS

1. To find the ______________________, if any, of the graph of an equation,

   let _______________ in the equation and solve for ________, where ______ is a real number.

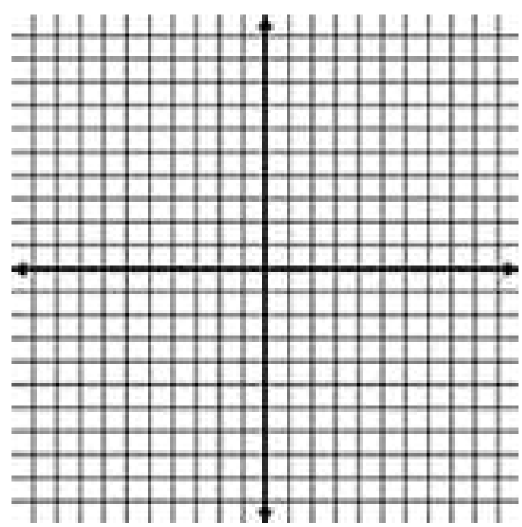
2. To find the ______________________, if any, of the graph of an equation,

   let _______________ in the equation and solve for ________, where ______ is a real number.

Example 1: Find the intercepts and graph each equation by plotting points.

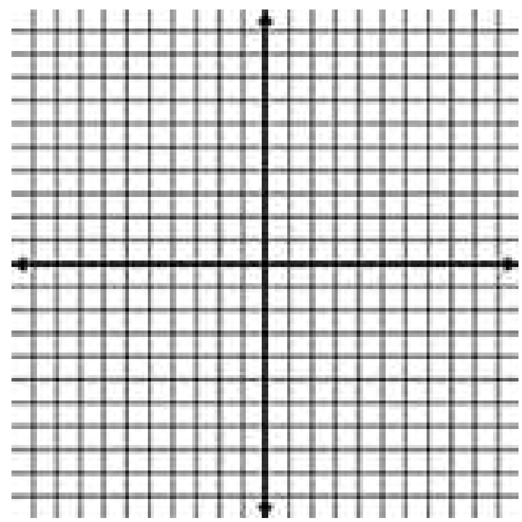
a. $y = x - 6$

x	$y = x - 6$	(x, y)



b. $4x^2 + y = 4$

x	$4x^2 + y = 4$	(x, y)

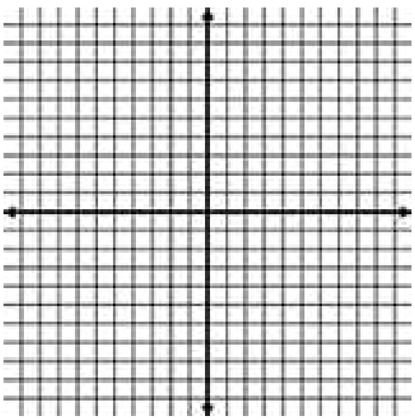


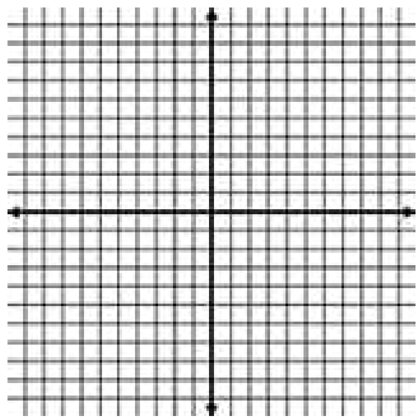
DEFINITION: SYMMETRY

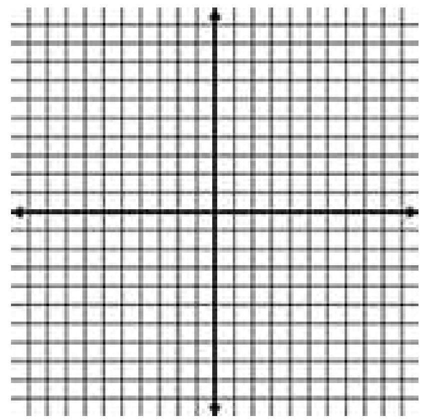
A graph is said to be symmetric with respect to the ________________ if, for every point __________ on the graph, the point ______________ is also on the graph.

A graph is said to be symmetric with respect to the ________________ if, for every point __________ on the graph, the point ______________ is also on the graph.

A graph is said to be symmetric with respect to the _________________ if, for every point ___________ on the graph, the point _______________ is also on the graph.







TESTS FOR SYMMETRY

____________________: Replace ________ by ________ in the equation. If an ____________________ equation results, the graph of the equation is ____________________ with respect to the ________________.

___________________: Replace ________ by ________ in the equation. If an ____________________ equation results, the graph of the equation is ____________________ with respect to the ________________.

___________________: Replace ________ by ________ and ________ by ________ in the equation. If an ____________________ equation results, the graph of the equation is ____________________ with respect to the _______________.

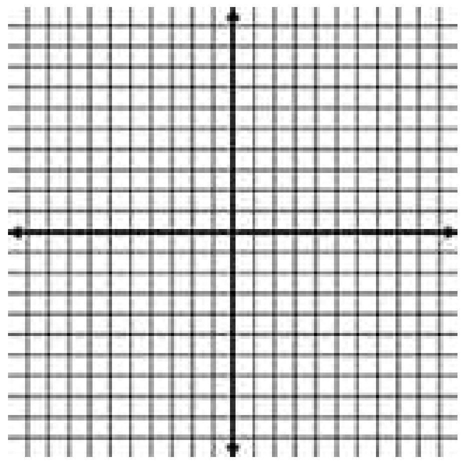
Example 2: Plot the point $(4,-2)$.

Plot the point that is symmetric to $(4,-2)$ with respect to the

a. x-axis

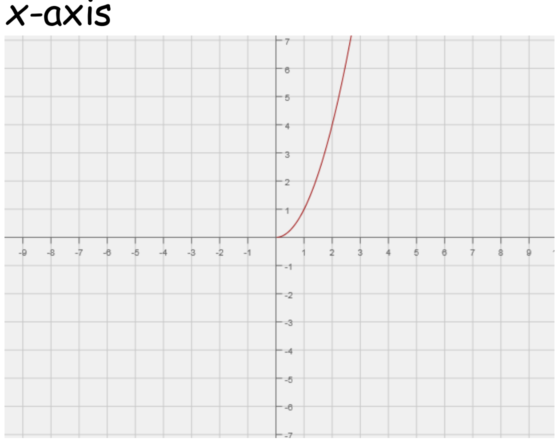
b. y-axis

c. origin

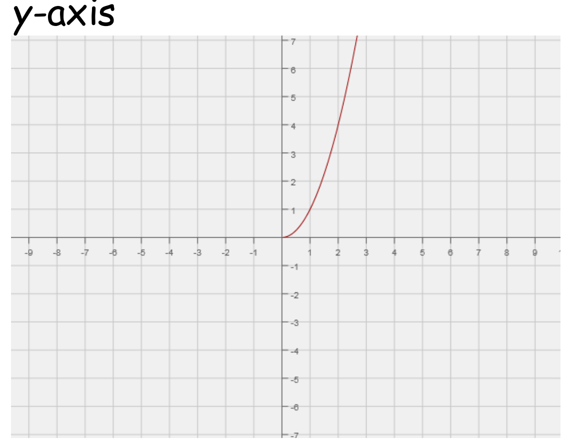


Example 3: Draw a complete graph so that it has the type of symmetry indicated.

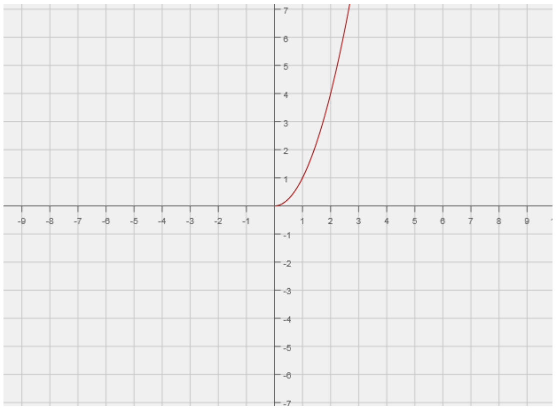
a. x-axis



b. y-axis



c. origin



Example 4: List the intercepts and test for symmetry.

a. $y^2 = x + 9$

b. $y = x^4 - 2x^2 - 8$

c. $y = \sqrt[5]{x}$

d. $y = \dfrac{x^4 + 1}{2x^5}$

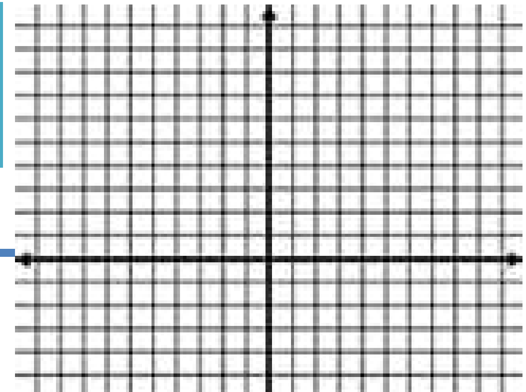
Example 5: If $(a,-5)$ is on the graph of $y = x^2 + 6x$, what is a?

KNOW HOW TO GRAPH KEY EQUATIONS

Example 6: Sketch the graph using intercepts and symmetry.

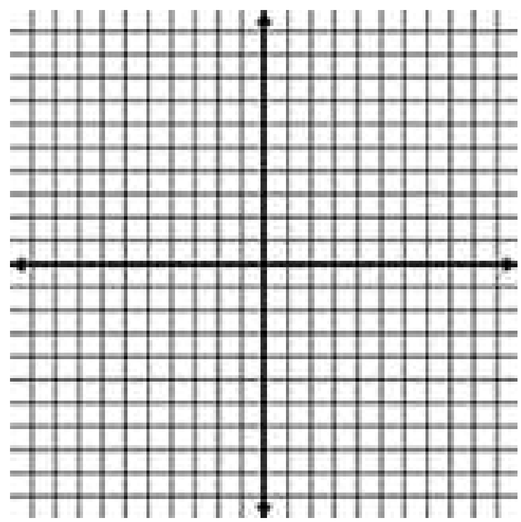
a. $y = \frac{1}{x}$

x	$y = \frac{1}{x}$	(x, y)



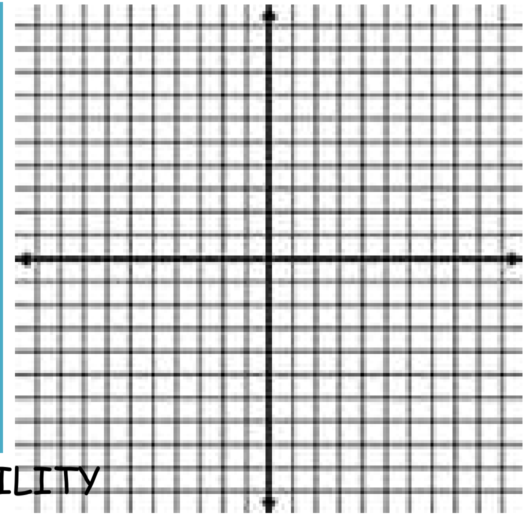
b. $x = y^2$

$x = y^2$	y	(x, y)



c. $y = x^3$

x	$y = x^3$	(x, y)



1.3: SOLVING EQUATIONS USING A GRAPHING UTILITY

When you are done with your homework, you should be able to…

- π Solve Equations Using a TI-83/TI-84 Graphing Calculator

Warm-up: Solve for *y*.

a. $-x - 8y = 7$

b. $x^3 - 2y = 6$

SOLVE EQUATIONS USING A TI-83/TI-84 GRAPHING CALCULATOR

When a graphing calculator is used to solve an equation, usually ____________________ solutions are obtained. Unless otherwise stated, we will approximate solutions as decimals rounded to ________ decimal places.

The __________________ or __________________ feature of a graphing calculator can be used to find the solutions of an equation when one side of the equation is _________. In using this feature to solve equations, we make use of the fact that when the graph of an equation in ________ variables, ______ and _______, crosses or touches the _________________ then _________________. For this reason, any value of ______ for which ___________ will be a _________________ to the equation. That is, solving an equation for _______ when one side of the equation is 0 is equivalent to finding where the graph of the corresponding equation in two variables crosses or touches the _______________.

STEPS FOR APPROXIMATING SOLUTIONS OF EQUATIONS USING ZERO OR ROOT

1. _________________ *y*. So you will have *y* = {expression in *x*}.
2. Graph ___________________________________.
3. Use ZERO or ROOT to determine each _______________________ of the graph.

Example 1: Use ZERO or ROOT to approximate the real solutions, if any, of each equation rounded to two decimal places.

a. $-3x^4 + 8x^2 - 2x - 9 = 0$

b. $x^3 - 8x + 1 = 0$

STEPS FOR APPROXIMATING SOLUTIONS OF EQUATIONS USING INTERSECT

1. Graph ____and

   graph ____
2. Use INTERSECT to determine the ____________________________ of each point in the intersection.

Example 2: Use ZERO or ROOT to approximate the real solutions, if any, of each equation rounded to two decimal places.

a. $-x^4 + 1 = 2x^2 - 3$

b. $\frac{1}{4}x^3 - 5x = \frac{1}{5}x^2 - 4$

Example 3: Solve each equation algebraically. Verify your solution using a graphing calculator.

a. $5-(2x-1)=10-x$

b. $\frac{4}{y}-5=\frac{18}{2y}$

c. $x^3+2x^2-9x-18=0$

1.4: LINES

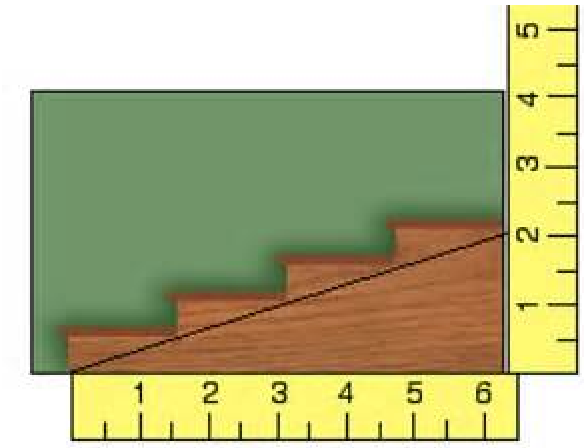
When you are done with your homework, you should be able to…

- π Calculate and Interpret the Slope of a Line
- π Graph Lines Given a Point and the Slope
- π Find the Equation of a Vertical Line
- π Use the Point-Slope Form of a Line; Identify Horizontal Lines
- π Find the Equation of a Line Given Two Points
- π Write the Equation of a Line in Slope Intercept Form
- π Identify the Slope and *y*-Intercept of a Line from Its Equation
- π Graph Lines Written in General Form Using Intercepts
- π Find Equations of Parallel Lines
- π Find Equations of Perpendicular Lines

Warm-up: Solve.

$$-5x+2(1-3x)=x-3$$

CALCULATE AND INTERPRET THE SLOPE OF A LINE



Consider the staircase to the left. Each step contains exactly the same horizontal

_________ and the same vertical

_________. The ratio of the rise to the run,

called the ___________, is a numerical

measure of the _____________________ of the staircase.

DEFINITION

Let ________________ and ________________ be two distinct points. If

______________, the _________________, ______, of the nonvertical line *L*

containing *P* and *Q*, is defined by the formula

If ________________, *L* is a ________________ line and the slope *m* of *L* is

___________________ (since this results in division by _______).

Example 1: Determine the slope of the line containing the given points.

a. $(4,2);(3,4)$

b. $(-1,1);(2,3)$

c. $(2,0);(2,2)$

SQUARE SCREENS

To get an undistorted view of slope, the same ____________ must be used on each axis. Most graphing calculators have a rectangular screen. Because of this, using the same interval for *x* and *y* will result in a distorted view. On most graphing calculators, you can obtain a square screen by setting the ratio of *x* to *y* at _______________.

Example 2: On the same square screen, graph the following equations:

$y_1 = 0$

$y_2 = \frac{1}{2}x$

$y_3 = x$

$y_4 = 4x$

Example 3: On the same square screen, graph the following equations:

$y_1 = 0$

$y_2 = -\frac{1}{2}x$

$y_3 = -x$

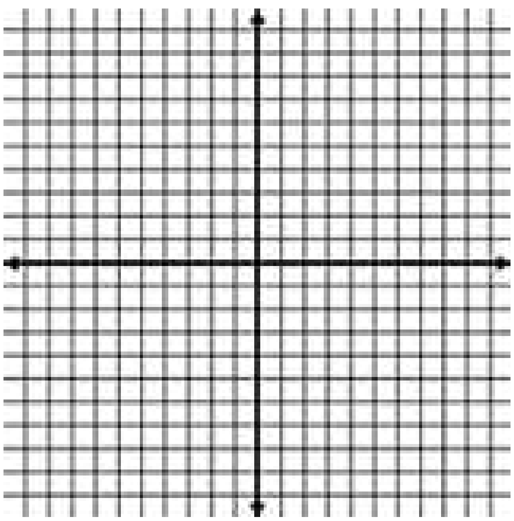
$y_4 = -4x$

What have we discovered???

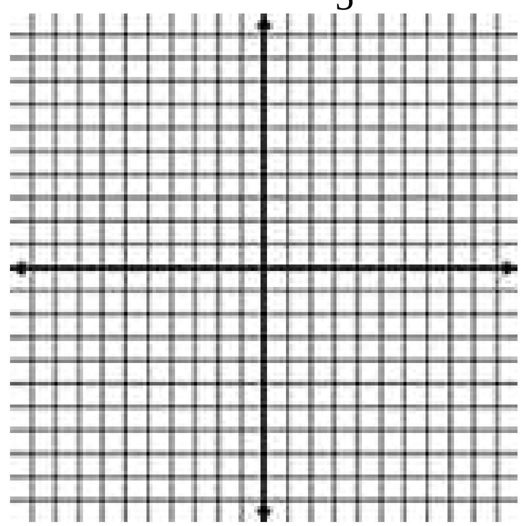
1. When the slope of a line is positive, the line slants __________________ from left to right.
2. When the slope of a line is negative, the line slants __________________ from left to right.
3. When the slope is zero, the line is ______________________.

Example 3: Graph the line containing the point *P* and having slope *m*. List two additional points that are on the line.

a. $P=(2,1); m=4$



b. $P=(1,3); m=-\frac{2}{5}$



EQUATION OF A VERTICAL LINE

A vertical line is given by an equation of the form

where ______ is the ________________.

POINT-SLOPE FORM OF A LINE

An equation of a nonvertical line with slope *m* that contains the point ________ is

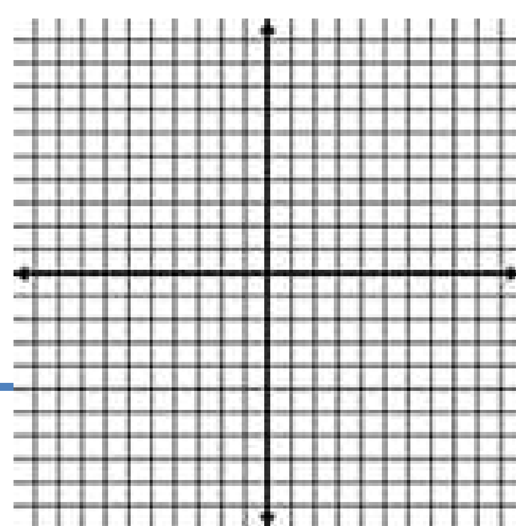
EQUATION OF A HORIZONTAL LINE

A horizontal line is given by an equation of the form

where _____ is the ____________________.

FINDING AN EQUATION OF A LINE GIVEN TWO POINTS

EXAMPLE 4: Find an equation of the line containing the points $(5,-1)$ and $(-6,8)$. Graph the line.



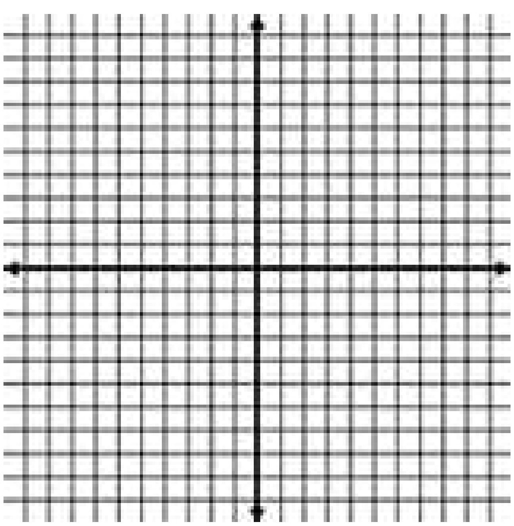
SLOPE-INTERCEPT FORM OF A LINE

An equation of a line with slope *m* and *y*-intercept *b* is

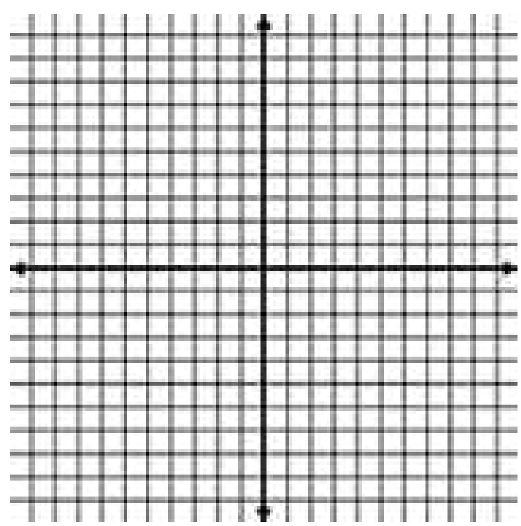
IDENTIFY THE SLOPE AND *y*-INTERCEPT OF A LINE GIVEN ITS EQUATION

EXAMPLE 5: Find the slope *m* and *y*-intercept *b* of the given equation. Graph the equation.

a. $y = 5x + 2$



b. $2x - 3y = 9$



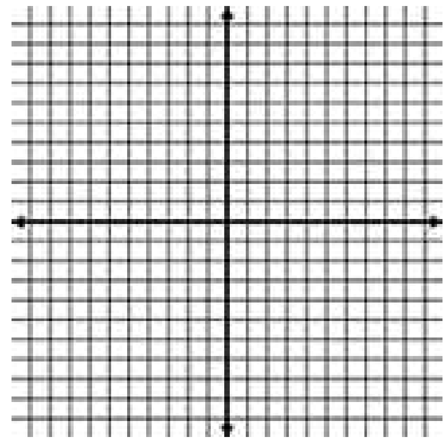
EQUATION OF A LINE IN GENERAL FORM

The equation of a line in general form is where ______, ______, and _____ are real numbers and *A* and *B* are not both zero.

GRAPHING AN EQUATION IN GENERAL FORM USING ITS INTERCEPTS

EXAMPLE 6: Graph the equation $-4x+2y=-12$ by finding its intercepts.

x	y	(x, y)



FIND EQUATIONS OF PARALLEL LINES

When two lines in the plane do not ______________________, they are said to be _____________________.

CRITERION FOR PARALLEL LINES

Two nonvertical lines are ______________________ if and only if their__________ are _________________ and they have different ____________________________.

SHOWING THAT TWO LINES ARE PARALLEL

EXAMPLE 7: Show that the lines given by the following equations are parallel.

$$y=-x+4$$
$$x+y=-1$$

FIND EQUATIONS OF PERPENDICULAR LINES

When two lines ____________________ at a_____________ angle __________, they are said to be ____________________.

CRITERION FOR PERPENDICULAR LINES

Two nonvertical lines are _____________________ if and only if the product of their _______________ is __________.

SHOWING THAT TWO LINES ARE PERPENDICULAR

EXAMPLE 8: Show that the lines given by the following equations are perpendicular.

$$y = \frac{1}{2}x - 10$$

$$y = -2x - \frac{1}{3}$$

FINDING THE EQUATION OF A LINE GIVEN INFORMATION

The following examples will guide you on how to find the equation of a line when you are given different types of information.

EXAMPLE 9: Find an equation for the line with the given properties. Express your answer using the general form and the slope-intercept form.

a. Slope = 2; containing the point $(4,-3)$.

b. Slope = undefined; containing the point $\left(\frac{1}{2},7\right)$.

c. x-intercept is -4; y-intercept is 4

d. Containing the points $(-3,4)$ and $(2,5)$.

e. Parallel to the line $x-2y=-5$; containing the point $(-5,1)$.

f. Perpendicular to the line $y=8$; containing the point $(3,4)$.

EXAMPLE 10: The equations of two lines are given. Determine if the lines are parallel, perpendicular, or neither.

a.

$$y = 4x + 5$$
$$y = -4x + 2$$

b.

$$y = \frac{1}{3}x - 3$$
$$y = -3x + 4$$

2.1: FUNCTIONS

When you are done with your homework, you should be able to…

- π Determine Whether a Relation Represents a Function
- π Find the Value of a Function
- π Find the Domain of a Function Defined by an Equation
- π Form the Sum, Difference, Product, and Quotient of Two Functions

WARM-UP: Find the value(s) of *x* for which the rational expression $\frac{x-1}{2x^2 - x - 10}$ is undefined.

DETERMINE WHETHER A RELATION REPRESENTS A FUNCTION

When the _______________ of one variable is ________________ to the value of a second variable, we have a ____________________. A relation is a _________________________ between two ________________. If _______ and

________ are two elements in these sets and if a relation exists between ______ and ______, then we say that ______ ______________________ to _______ or that _______ ________________________ on ______, and we write __________________.

Relations can be expressed as an ________________________, _________________, and/or a _________________.

Example 1: Find the domain and range of the relation.

VEHICLE	NUMBER OF WHEELS
CAR	4
MOTORCYCLE	2
BOAT	0

DEFINITION OF A FUNCTION

Let _____ and _____ represent two nonempty sets. A __________________ from ______ into ______ is a relation that associates with each _________________ of _______ exactly _________ element of _______.

FUNCTIONS AS EQUATIONS AND FUNCTION NOTATION

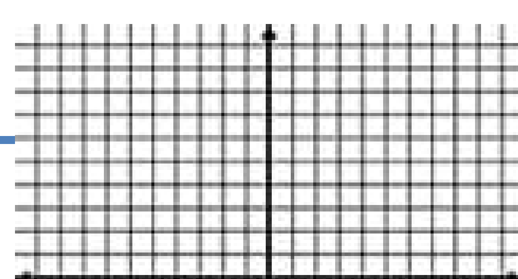
Functions are often given in terms of _________________ rather than as ________ of __________________ ________________. Consider the equation below, which describes the position of an object, in feet, dropped from a height of 500 feet after x seconds.

$$y = -16x^2 + 500$$

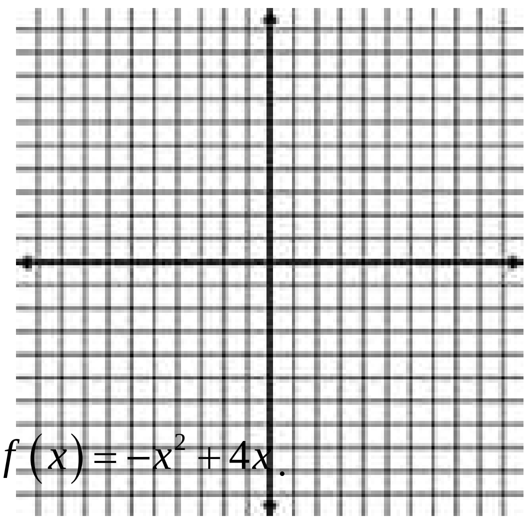
The variable ____ is a ______________ of the variable _____. For each value of x, there is one and only one value of _____. The variable x is called the _______________ variable because it can be _______________ any value from the _______________. The variable y is called the _______________ variable because its value ______________ on x. When an ________________ represents a ________________, the function is often named by a letter such as f, g, h, F, G, or H. Any letter can be used to name a function. The domain is the ______ of the function's ______________ and the range is the ______ of the function's _______________. If we name our function _____, the input is represented by _____, and the output is represented by ______. The notation ______ is read " ____ of ____" or "____ at ____. So we may rewrite $y = -16x^2 + 500$ as _____________________. Now let's evaluate our function after 1 second:

Example 2: Determine whether each relation represents a function. Then identify the domain and range.

a. $\{(-6,1),\ (-1,1),\ (0,1),\ (1,1),\ (2,1)\}$



b. $\{(3,3),\ (-2,0),\ (4,0),\ (-2,-5)\}$



Example 3: Find the indicated function values for $f(x) = -x^2 + 4x$.

a. $f(4)$

b. $3f(-2)$

c. $f(x+1)$

d. $\dfrac{f(x+h)-f(x)}{h},\ h\neq 0$

Example 4: Find the indicated function and domain values using the table below.

a. $h(-2)$

b. $h(1)$

c. For what values of x is $h(x)=1$?

x	$h(x)$
-2	2
-1	1
0	0
1	1
2	2

Example 5: Determine if the following equations define *y* as a function of *x*.

a. $xy=5$

b. $x^2+y^2=16$

FINDING VALUES OF A FUNCTION ON A CALCULATOR

Example 6: Let $f(x) = -x^3 - x + 2$.Use a graphing calculator to find the following values:

a. $f(4)$

b. $f(-2)$

STEPS FOR FINDING THE DOMAIN OF A FUNCTION DEFINED BY AN EQUATION

1. Start with the domain as the set of ________________ numbers.
2. If the equation has a denominator, ____________________ any numbers that give a _______________ denominator.
3. If the equation has a ___________________ of even ___________________, exclude any numbers that cause the expression inside the radical to be _______________________.

Example 7: Find the domain of each of the following functions.

a. $h(x) = \sqrt{2x-1}$

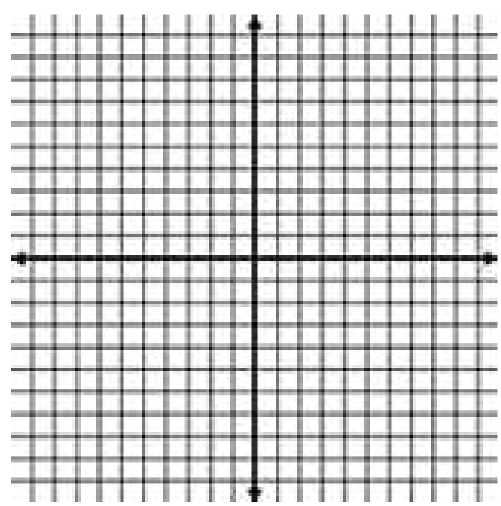
b. $g(x)=\dfrac{8x}{x^2-81}$

THE ALGEBRA OF FUNCTIONS

Consider the following two functions:

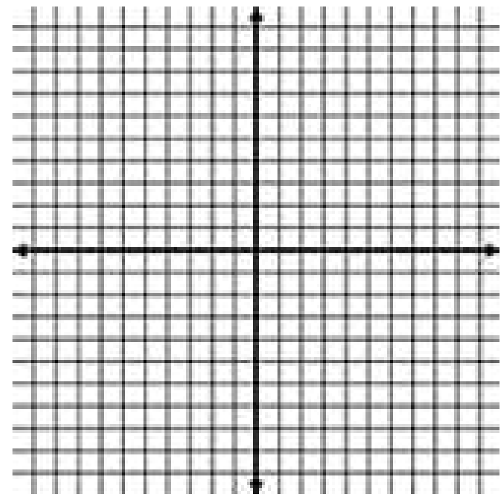
$f(x)=2x$ and $g(x)=x-1$

Let's graph these two functions on the same coordinate plane.



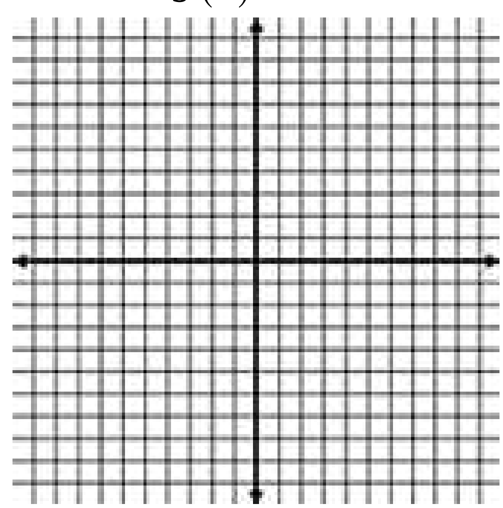
Now find and graph the sum of f and g.

$(f+g)(x)=$



Now find and graph the difference of f and g. $f(x)=2x$ and $g(x)=x-1$

$(f-g)(x)=$



Now find and graph the product of f and g on your graphing calculator.

$(fg)(x)=$

Now find and graph the quotient of f and g on your graphing calculator.

$\left(\frac{f}{g}\right)(x) =$

THE ALGEBRA OF FUNCTIONS: SUM, DIFFERENCE, PRODUCT, AND QUOTIENT OF FUNCTIONS

Let f and g be two functions. The ______ $f+g$, the _______________ $f-g$, the ______________ fg, and the ______________ $\frac{f}{g}$ are ______________ whose domains are the set of all real numbers _________________ to the domains of f and g, defined as follows:

1. Sum: _________________________________
2. Difference: ___________________________
3. Product: _______________________________
4. Quotient: ______________________________, provided _________________

Example 8: Let $f(x)=x^2+4x$ and $g(x)=2-x$. Find the following:

a. $(f+g)(x)$

d. $(fg)(x)$

b. $(f+g)(4)$

e. $(fg)(3)$

c. $f(-3)+g(-3)$

f. The domain of $\left(\frac{f}{g}\right)(x)$

2.2: THE GRAPH OF A FUNCTION

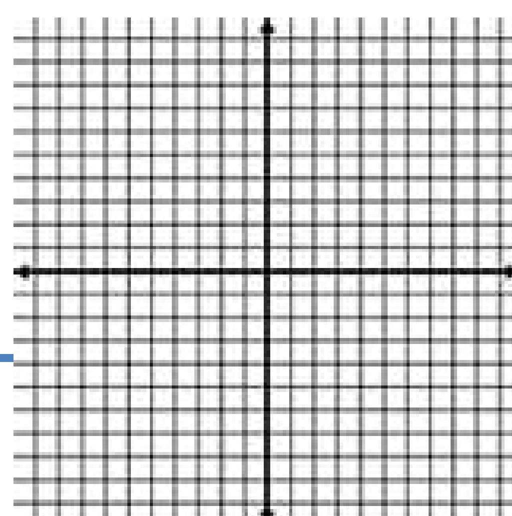
When you are done with your homework, you should be able to…

- π Identify the Graph of a Function
- π Obtain Information from or about the Graph of a Function

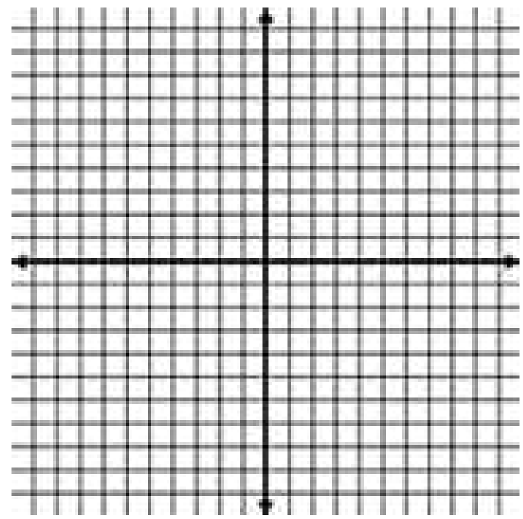
WARM-UP:

Graph the following equations by plotting points.

a. $y=x^2$



b. $y = 3x - 1$

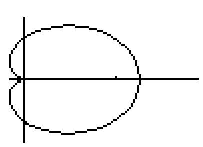


THE VERTICAL LINE TEST FOR FUNCTIONS

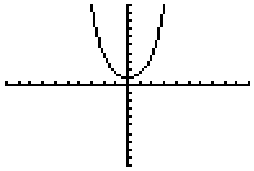
If any vertical line __________________ a graph in more than __________ point, the graph _________ __________ define ____ as a function of ____.

Example 1: Determine whether the graph is that of a function.

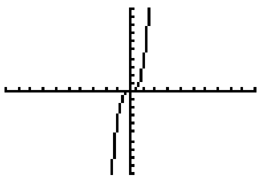
a.



b.



c.



OBTAINING INFORMATION FROM GRAPHS

You can obtain information about a function from its graph. At the right or left of a graph, you will often find ___________ dots, ___________ dots, or __________.

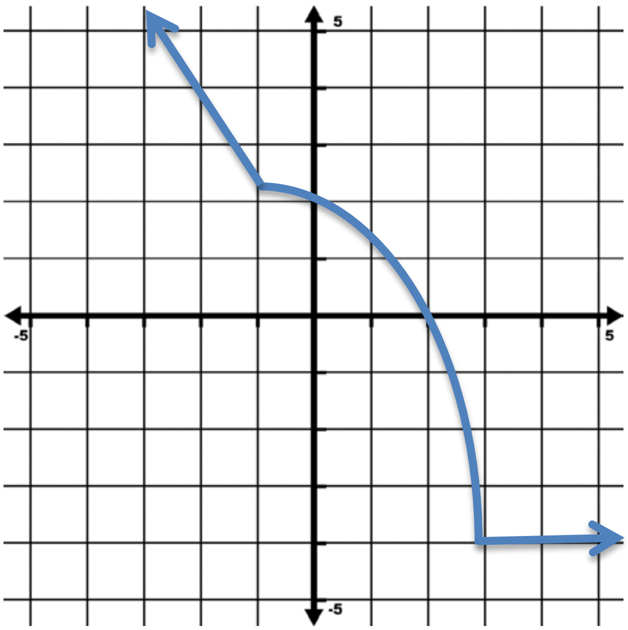
- π A closed dot indicates that the graph does not ___________ beyond this point and the ____________ belongs to the ______________
- π An open dot indicates that the graph does not ___________ beyond this point and the ____________ DOES NOT belong to the ______________
- π An arrow indicates that the graph extends ________________ in the direction in which the arrow ________________

REVIEWING INTERVAL NOTATION

INTERVAL NOTATION	SET-BUILDER NOTATION	GRAPH
(a,b)		x
$[a,b]$		x
$[a,b)$		x
$(a,b]$		x
(a,∞)		x
$[a,\infty)$		x
$(-\infty,b)$		x
$(-\infty,b]$		x
$(-\infty,\infty)$		x

Example 2: Use the graph of f to determine each of the following.

f



a. $f(0)$

b. $f(-2)$

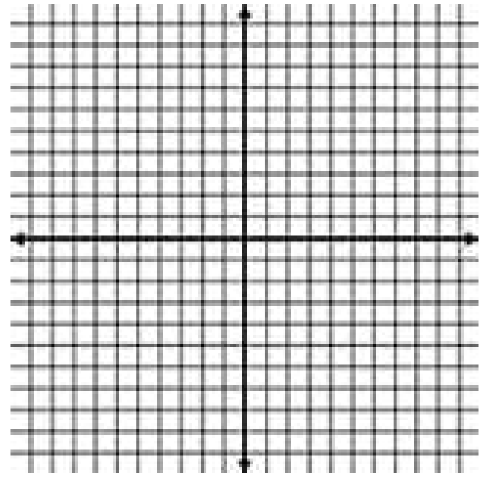
c. For what value of x is $f(x) = 3$?

d. The domain of f

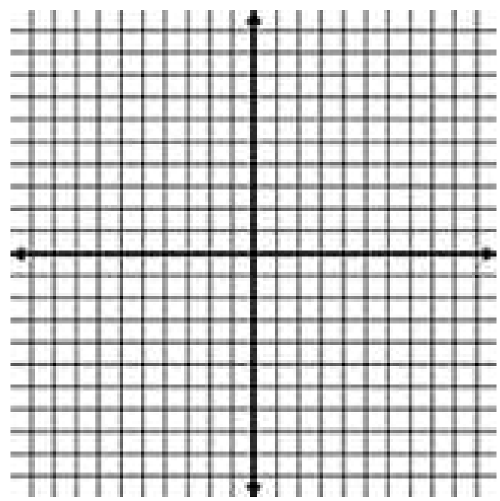
e. The range of f

Example 3: Graph the following functions by plotting points and identify the domain and range.

a. $f(x) = -x - 2$



b. $H(x) = x^2 + 1$



Example 4: Consider the function $f(x) = \dfrac{x^2 + 2}{x + 4}$.

a. Is the point $\left(1,\frac{3}{5}\right)$ on the graph?

b. If $x=0$, what is $f(x)$? What point is on the graph of f?

c. If $f(x)=\frac{1}{2}$, what is x? What point(s) are on the graph of f?

d. What is the domain of f?

e. List the *x*-intercepts, if any, of the graph of f.

f. List the *y*-intercepts, if any, of the graph of f.

APPLICATION

If an object weighs *m* pounds at sea level, then its weight *W*, in pounds, at a height of *h* miles above sea level is given approximately by

$$W(h) = m\left(\frac{4000}{4000+h}\right)^2$$

a. If Amy weighs 120 pounds at sea level, how much will she weigh on Pike's Peak, which is 14,110 feet above sea level?

b. Use a graphing calculator to graph the function $W = W(h)$.

c. Create a TABLE with $\text{TblStart} = 0$ and $\Delta\text{Tbl} = 0.5$ to see how the weight W varies as h changes from 0 to 5 miles.

d. At what height will Amy weigh 119.95 pounds?

e. Does your answer to part d seem reasonable? Explain.

When you are done with your homework you should be able to…

- π Determine Even and Odd functions from a Graph
- π Identify Even and Odd functions from the Equation
- π Use a Graph to Determine Where a Function is Increasing, Decreasing, or Constant
- π Use a Graph to Locate Local Maxima and Local Minima
- π Use a Graph to Locate the Absolute Maximum and Absolute Minimum
- π Use a Graphing Utility to Approximate Local Maxima and Local Minima
- π Find the Average Rate of Change of a Function

WARM-UP: Test the equation $y = -x^2 + 3$ for symmetry with respect to the *x*-axis, *y*-axis, and the origin.

EVEN FUNCTIONS

A function f is __________________ if, for every number ____ in its domain, the number _____ is also in the domain and

ODD FUNCTIONS

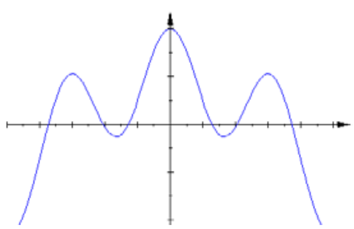
A function f is ___________________ if, for every number _____ in its domain, the number ______ is also in the domain and

THEOREM

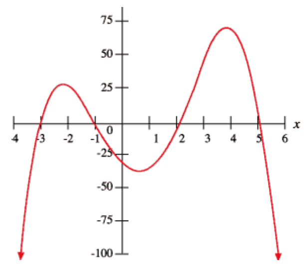
A function is ____________ if and only if its graph is symmetric with respect to the _____________________. A function is _____________ if and only if its graph is symmetric with respect to the ___________________.

Example 1: Determine whether each graph given below is the graph of an even function, an odd function, or a function that is neither even nor odd.

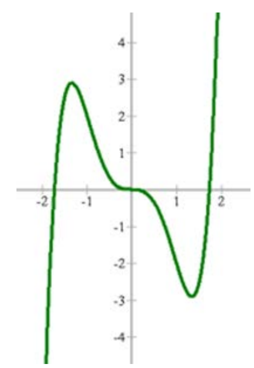
a.



b.



c.



Example 2: Determine algebraically whether each function is even, odd, or neither.

a. $h(x) = 3x^3 + 5$

b. $F(x)=\frac{2x}{|x|}$

c. $f(x)=2x^4-x^2$

INCREASING/DECREASING/CONSTANT INTERVALS OF A FUNCTION

A function f is ___________________ on an open _________________ _____if, for any choice of ______ and ______ in I, with ____________, we have _______________.

A function f is ___________________ on an open _________________ _____if, for any choice of ______ and ______ in I, with ____________, we have _______________.

A function f is ___________________ on an open _________________ _____if, for all choices of ______ in I, the values of _______ are ____________.

**NOTE: We describe the behavior of a graph in terms of its ______________!!!

LOCAL EXTREMA

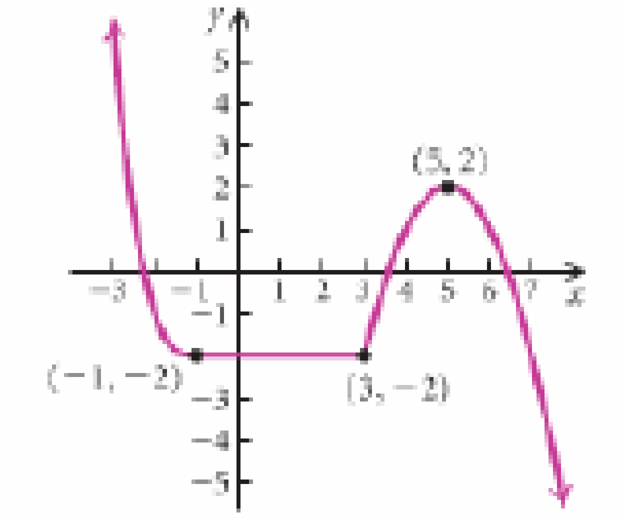
A function f has a ___________ ______________ at _____ if there is an open interval I containing c so that for all x in I, __________________. We call ________ a ____________ _________________ ____________ of ____.

A function f has a ___________ ______________ at _____ if there is an open interval I containing c so that for all x in I, __________________. We call ________ a ____________ _________________ ____________ of ____.

**NOTE: The word ___________ is used to suggest that it is only near ____, that is, in some open interval containing c, that the value of ________ has these properties.

**NOTE: The _______________ is the local maximum or minimum value and it *occurs* at some ________________.

Example 3: Consider the graph of the function given below.



a. On what interval(s) is f increasing?

b. On what interval(s) is f decreasing?

c. On what interval(s) is f constant?

d. List the local minima.

e. List the ordered pair(s) where a local minimum occurs.

f. List the local maxima.

g. List the ordered pair(s) where a local maximum occurs.

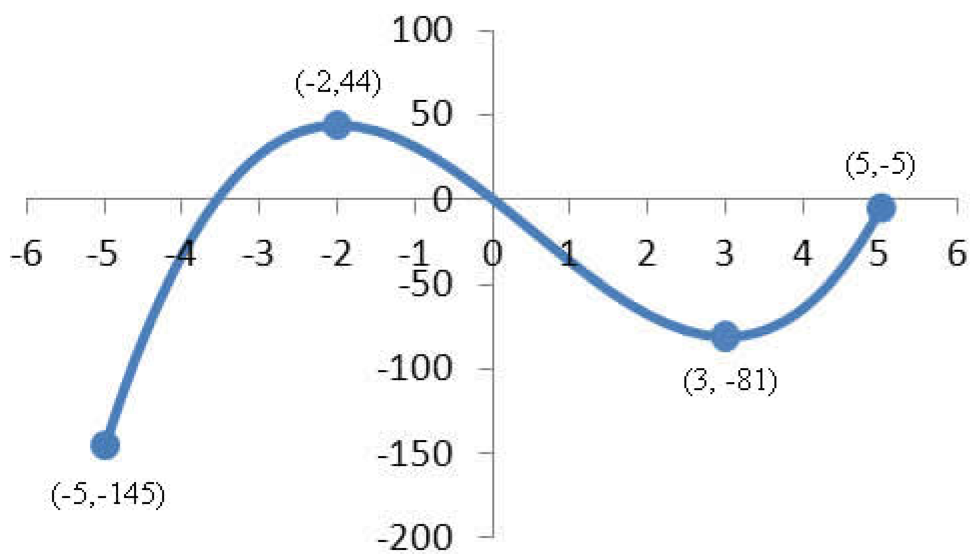
ABSOLUTE EXTREMA

Let f denote a function defined on some interval I. If there is a number ______ in I for which ______________ for all x in I, then ______ is the _______________ _________________ of ______ on _____ and we say the _______________ _______________ of ______ occurs at ______.

If there is a number ______ in I for which ______________ for all x in I, then ______ is the _______________ _________________ of ______ on _____ and we say the _______________ _______________ of ______ occurs at ______.

Example 4: Find the absolute minimum and the absolute maximum, if they exist, of the following graphs below.

a.



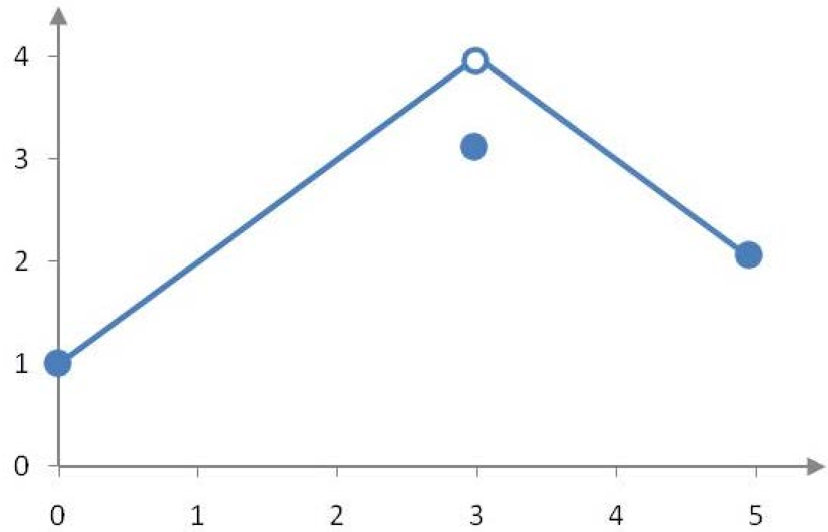
The absolute minimum is _________________________.

The absolute minimum occurs at _________________________.

The absolute maximum is _________________________.

The absolute maximum occurs at _________________________.

b.



The absolute minimum is ________________________.

The absolute minimum occurs at ________________________.

The absolute maximum is ________________________.

The absolute maximum occurs at ________________________.

EXTREME VALUE THEOREM

If f is a continuous function whose domain is a closed interval $[a,b]$, then f has an ____________________ ________________ and an ____________________ ______________________ on $[a,b]$.

**NOTE: You can consider a continuous function to be a function whose graph has no ____________ or ______________ and can be __________________ without lifting the pencil from the paper.

AVERAGE RATE OF CHANGE

If _____ and _____, _______________, are in the domain of a function $y = f(x)$, the

____________________ ___________________ of _________________ from _____ to

______ is defined as

Average rate of change =

Example 5: Find the average rate of change of $f(x) = -x^3 + 1$

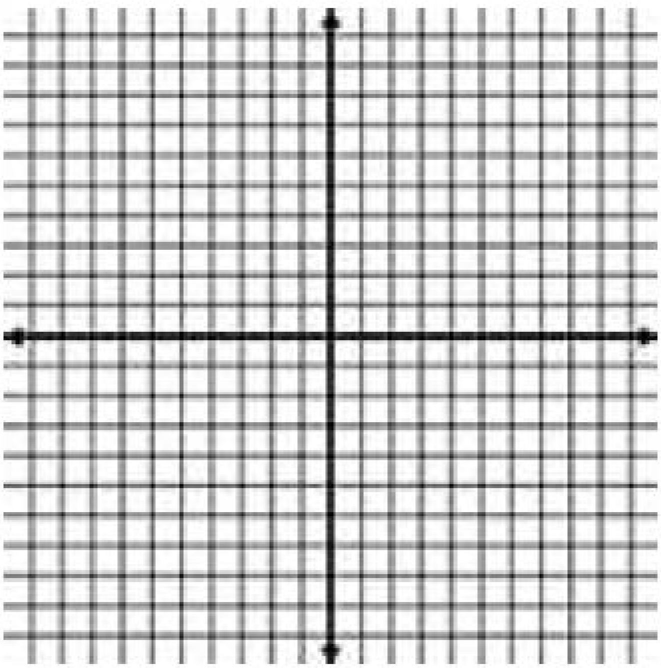
a. From 0 to 2

b. From 1 to 3

c. From -1 to 1

THEOREM: SLOPE OF THE SECANT LINE

The _________________ ______________ of _______________ of a function from ______ to _____ equals the _____________ of the _____________ line containing the two points ____________ and ___________ on its graph.



Example 6: Consider $h(x) = -2x^2 + x$

Find an equation of the secant line containing the *x*-coordinates 0 and 3.

2.4: LIBRARY OF FUNCTIONS; PIECEWISE-DEFINED FUNCTIONS

When you are done with your homework, you should be able to…

- π Graph the Functions Listed in the Library of Functions
- π Graph Piecewise-defined Functions

WARM-UP: Consider $f(x) = x^4 - 3$

a. What is the average rate of change from -1 to 2.

b. Find an equation of the secant line containing the *x*-coordinates -1 and 2.

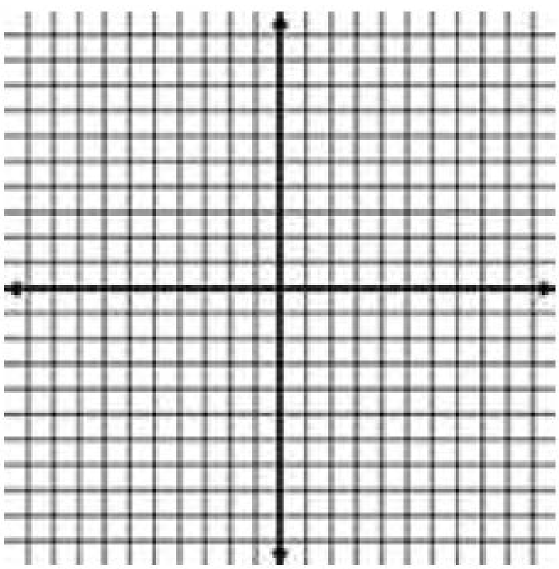
THE LIBRARY OF FUNCTIONS

Example 1: Consider the function $f(x)=b$.

a. Determine whether $f(x)=b$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=b$.

c. Graph $f(x)=b$ by hand.



PROPERTIES OF $f(x)=b$

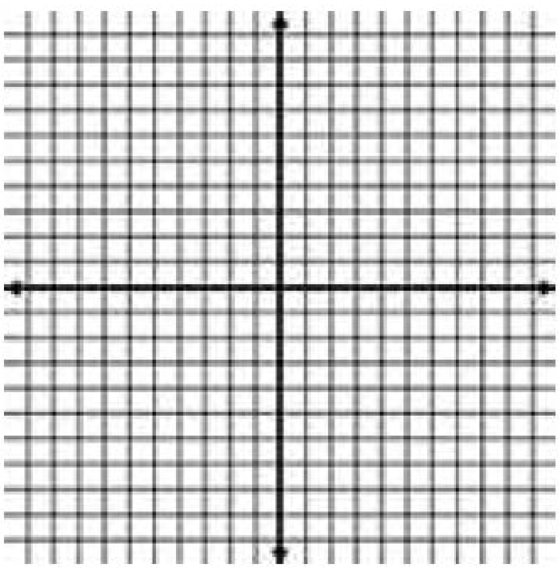
1. The domain is the set of_____________ numbers. The range of f is the set consisting of a single number _____.
2. The *y*-intercept of the graph of $f(x)=b$ is ______.
3. The graph is a _______________ line. The function is ___________________ with respect to the ________________. The function is ______________.

Example 2: Consider the function $f(x)=x$.

a. Determine whether $f(x)=x$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=x$.

c. Graph $f(x)=x$ by hand.



PROPERTIES OF $f(x)=x$

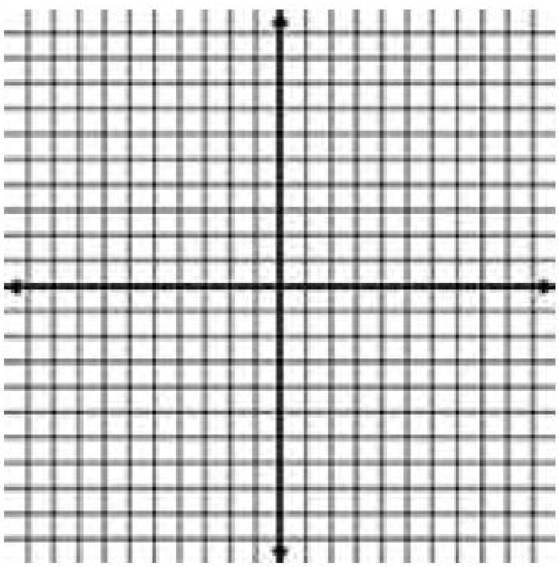
1. The domain and range are the set of_____________ numbers.
2. The *x*-intercept of the graph of $f(x)=x$ is ____. The *y*-intercept of the graph of $f(x)=x$ is ______.
3. The graph is _______________ with respect to the _________________.
4. The function is _______________.
5. The function is ______________________ on the interval _____________

Example 3: Consider the function $f(x) = x^2$.

a. Determine whether $f(x) = x^2$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = x^2$.

c. Graph $f(x) = x^2$ by hand.



PROPERTIES OF $f(x) = x^2$

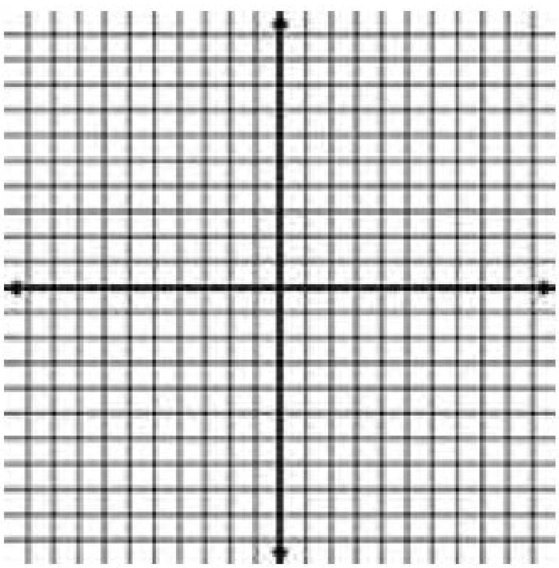
1. The domain is the set of_____________ numbers. The range is the set of _______________________ real numbers.
2. The *x*-intercept of the graph of $f(x) = x^2$ is _____. The *y*-intercept of the graph of $f(x) = x^2$ is _______.
3. The graph is ________________ with respect to the __________________.
4. The function is ________________.
5. The function is _______________________ on the interval ______________ and _______________________ on the interval ______________.

Example 4: Consider the function $f(x)=x^3$.

a. Determine whether $f(x)=x^3$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=x^3$.

c. Graph $f(x)=x^3$ by hand.



PROPERTIES OF $f(x)=x^3$

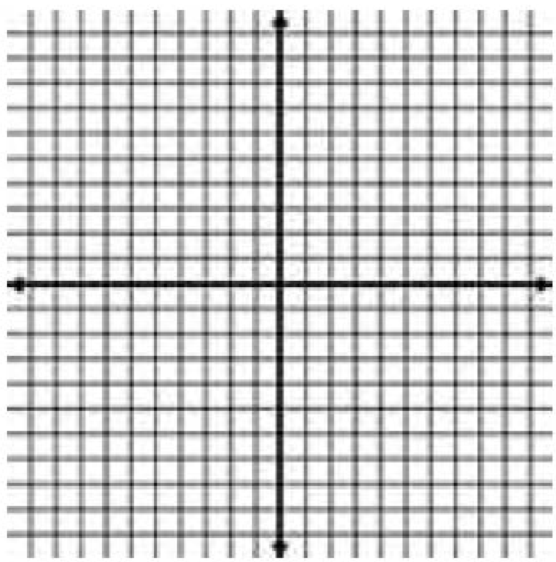
1. The domain and range are the set of______________ numbers.
2. The *x*-intercept of the graph of $f(x)=x^3$ is _____. The *y*-intercept of the graph of $f(x)=x^3$ is _______.
3. The graph is _________________ with respect to the ___________________.
4. The function is _________________.
5. The function is _________________________ on the interval _______________.

Example 5: Consider the function $f(x)=\sqrt{x}$.

a. Determine whether $f(x)=\sqrt{x}$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=\sqrt{x}$.

c. Graph $f(x)=\sqrt{x}$ by hand.



PROPERTIES OF $f(x)=\sqrt{x}$

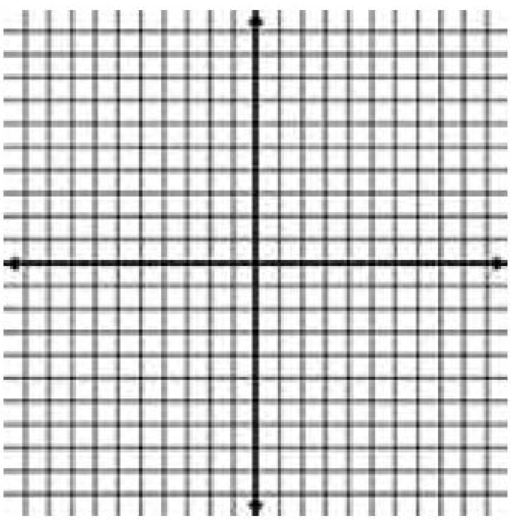
1. The domain and range are the set of _________________________ ______________ numbers.
2. The *x*-intercept of the graph of $f(x)=\sqrt{x}$ is _____. The *y*-intercept of the graph of $f(x)=\sqrt{x}$ is _______.
3. The function is _________________ ______________ nor _______________.
4. The function is _________________________ on the interval _______________.
5. The function has an ___________________ _____________________ of ______ at ________________.

Example 6: Consider the function $f(x)=\sqrt[3]{x}$.

a. Determine whether $f(x)=\sqrt[3]{x}$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=\sqrt[3]{x}$.

c. Graph $f(x)=\sqrt[3]{x}$ by hand.



PROPERTIES OF $f(x)=\sqrt[3]{x}$

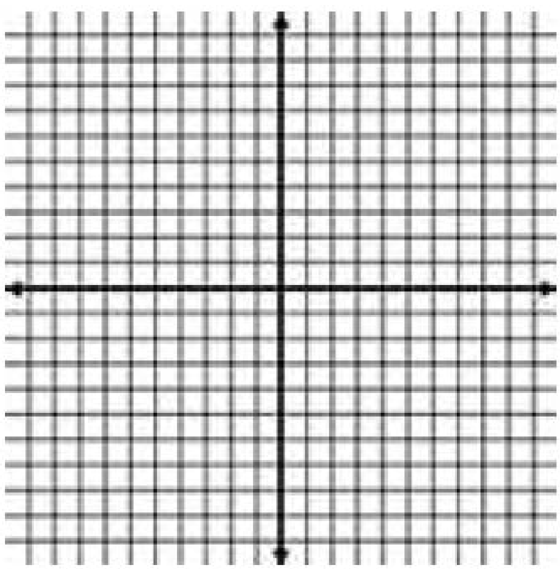
1. The domain and range are the set of______________ numbers.
2. The *x*-intercept of the graph of $f(x)=\sqrt[3]{x}$ is _____. The *y*-intercept of the graph of $f(x)=\sqrt[3]{x}$ is _______.
3. The graph is ________________ with respect to the __________________. The function is ________________.
4. The function is _______________________ on the interval ______________.
5. The function does not have any local _________________ or local _________________.

Example 7: Consider the function $f(x)=\frac{1}{x}$.

a. Determine whether $f(x)=\frac{1}{x}$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=\frac{1}{x}$.

c. Graph $f(x)=\frac{1}{x}$ by hand.



PROPERTIES OF $f(x)=\frac{1}{x}$

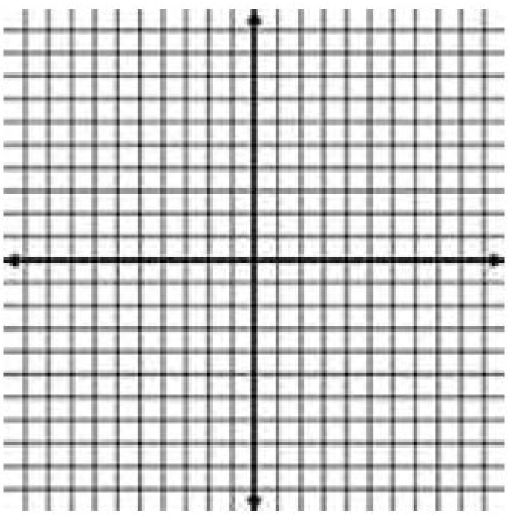
1. The domain and range are the set of all_______________ real numbers.
2. The graph of $f(x)=\frac{1}{x}$ has _____intercepts.
3. The graph is _________________ with respect to the ___________________.
4. The function is _________________.
5. The function is _________________________ on the interval _______________ and _________________________ on the interval _______________.

Example 8: Consider the function $f(x)=|x|$.

a. Determine whether $f(x)=|x|$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=|x|$.

c. Graph $f(x)=|x|$ by hand.



PROPERTIES OF $f(x)=|x|$

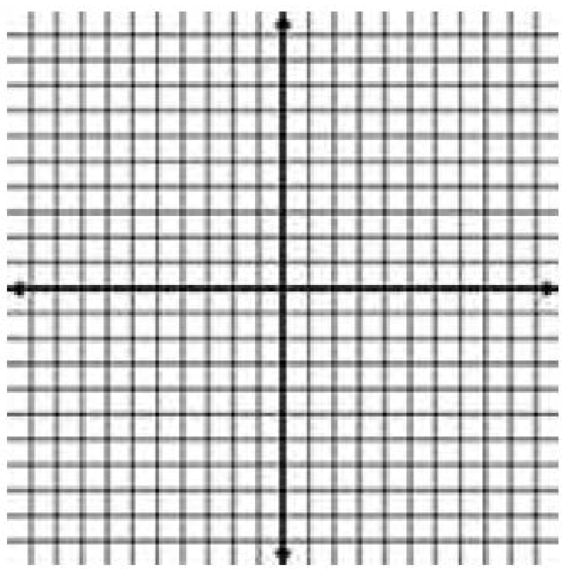
1. The domain is the set of_______________ numbers. The range of f is _____________________.
2. The *x*-intercept of the graph of $f(x)=|x|$ is _____. The *y*-intercept of the graph of $f(x)=|x|$ is _______.
3. The graph is __________________ with respect to the ____________________.
   The function is __________________.
4. The function is _________________________ on the interval ________________ and _______________________ on the interval __________________.
5. The function has an _________________ _________________ of _____ at _______.

Example 9: Consider the function $f(x)=\text{int}(x)$.

a. Determine whether $f(x)=\text{int}(x)$ is even, odd, or neither. State whether the graph is symmetric with respect to the *y*-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x)=\text{int}(x)$.

c. Graph $f(x)=\text{int}(x)$ by hand.

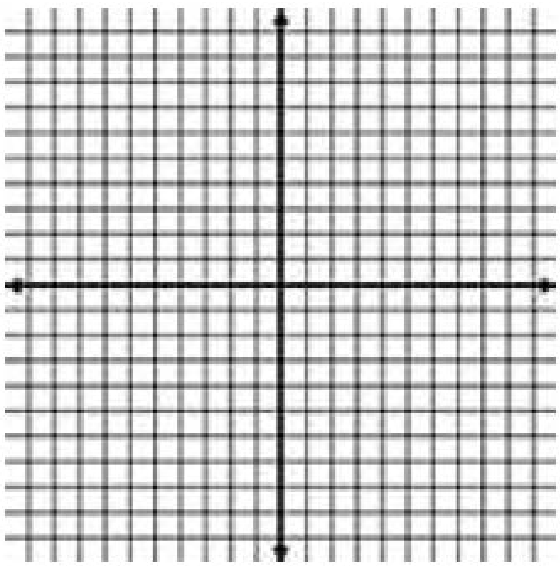


PROPERTIES OF $f(x)=\text{int}(x)$

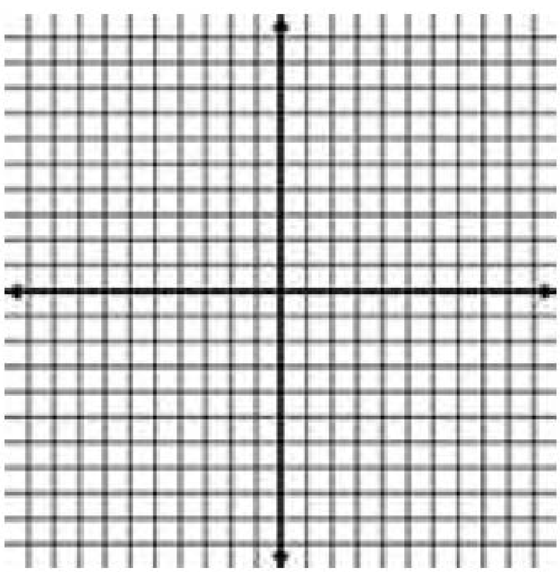
1. The domain is the set of all_____________ numbers. The range is the set of __________________.
2. The *x*-intercepts lie on the interval __________. The *y*-intercept is _______.
3. The function is ________________ __________ nor _________.
4. The function is ______________________ on every interval of the form _____________________, for _____ an _____________________.

Example 10: Sketch the graph of the following functions. Find the domain of each function. Locate any intercepts. Based on the graph, find the range. Is f continuous on its domain?

a. $f(x)=\begin{cases} -3x & \text{if } x<-1 \\ 0 & \text{if } x=1 \\ 2x^2+1 & \text{if } x>1 \end{cases}$



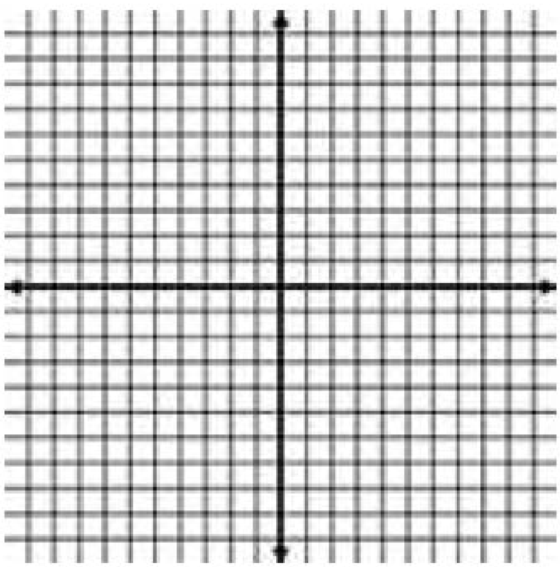
b. $f(x)=\begin{cases} 2-x & \text{if } -3\le x<1 \\ \sqrt{x} & \text{if } x>1 \end{cases}$



APPLICATION

The short-term (no more than 24 hours) parking fee F (in dollars) for parking *x* hours at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x)=\begin{cases} 2 & \text{if } 0<x\le 1 \\ 4 & \text{if } 1<x\le 3 \\ 10 & \text{if } 3<x\le 4 \\ 5\operatorname{int}(x+1)+2 & \text{if } 4<x<9 \\ 51 & \text{if } 9\le x\le 24 \end{cases}$$



Determine the fee for parking in the short-term parking garage for

a. 2 hours

b. 7 hours

c. 15 hours

d. 8 hours and 24 minutes

2.5: GRAPHING TECHNIQUES: TRANSFORMATIONS

When you are done with your homework, you should be able to…

- π Graph Functions Using Vertical and Horizontal Shifts
- π Graph Functions Using Compressions and Stretches
- π Graph Functions Using Reflections about the *x*-axis or *y*-axis

WARM-UP:

1. Consider the functions

$$Y_1 = x^3$$
$$Y_2 = x^3 + 4$$
$$Y_3 = x^3 - 4$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1, Y_2, and Y_3.

c. Describe Y_2 in terms of Y_1.

d. Describe Y_3 in terms of Y_1.

2. Consider the functions

$$Y_1 = x^3$$
$$Y_2 = (x-4)^3$$
$$Y_3 = (x+4)^3$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1, Y_2, and Y_3.

c. Describe Y_2 in terms of Y_1.

d. Describe Y_3 in terms of Y_1.

3. Consider the functions

$$Y_1 = x^4$$
$$Y_2 = 2x^4$$
$$Y_3 = \frac{1}{2}x^4$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1, Y_2, and Y_3.

c. Describe Y_2 in terms of Y_1.

d. Describe Y_3 in terms of Y_1.

4. Consider the functions

$$Y_1 = x^4$$

$$Y_2 = -x^4$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1 and Y_2.

c. Describe Y_2 in terms of Y_1.

5. Consider the functions

$$Y_1 = \sqrt{x}$$
$$Y_2 = \sqrt{-x}$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1 and Y_2.

c. Describe Y_2 in terms of Y_1.

SUMMARY OF GRAPHING TECHNIQUES

TO GRAPH:	DRAW THE GRAPH OF f AND:	FUNCTIONAL CHANGE TO $f(x)$
VERTICAL SHIFTS		
$y = f(x) + k, \quad k > 0$	____________ the graph of f by ________ units.	_____ k to $f(x)$.
$y = f(x) - k, \quad k > 0$	____________ the graph of f by ________ units.	_________ k from $f(x)$.
HORIZONTAL SHIFTS		
$y = f(x+h), \quad h > 0$	____________ the graph of f to the __________ _____ units.	_____________ x by_________.
$y = f(x-h), \quad h > 0$	____________ the graph of f to the __________ _____ units.	_____________ x by_________.
COMPRESSING OR STRETCHING		
$y = af(x), \quad a > 0$	____________ each _____________ of $y = f(x)$ by _____. ___________ the graph of f ___________________ if $a > 1$. ___________ the graph of f __________________ if $0 < a < 1$.	_____________ $f(x)$ by_______.

$y = f(ax), \quad a > 0$	____________ each ______________ of $y = f(x)$ by ________. ___________ the graph of f ___________________ if $0 < a < 1$. ___________ the graph of f ___________________ if $a > 1$.	______________ x by________.

REFLECTION ABOUT THE *x*-AXIS

$y = -f(x)$	___________________ the graph of f about the ____________________.	______________ $f(x)$ by________.

REFLECTION ABOUT THE *y*-AXIS

$y = f(-x)$	___________________ the graph of f about the ____________________.	______________ x by________.

Example 1: Write the function whose graph is $y = x^2$, but is

a. Shifted to the left 8 units.

b. Shifted down 8 units.

c. Reflected about the *x*-axis.

d. Shifted to the up 8 units.

e. Vertically compressed by a factor of 8.

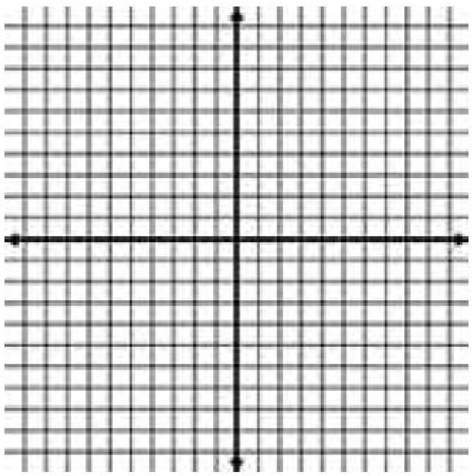
f. Horizontally stretched by a factor of 8 units.

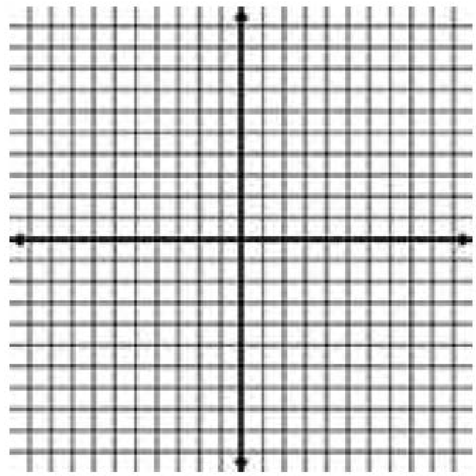
g. Shifted to the right 8 units.

e. Reflected about the *y*-axis.

Example 2: Graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages. Be sure to show at least three key points. Find the domain and range of each function.

a. $h(x)=\sqrt{x+1}$

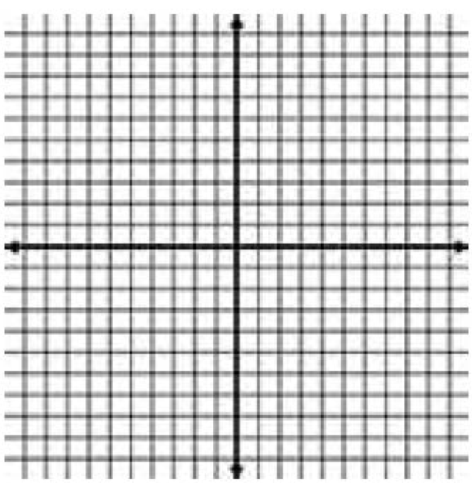


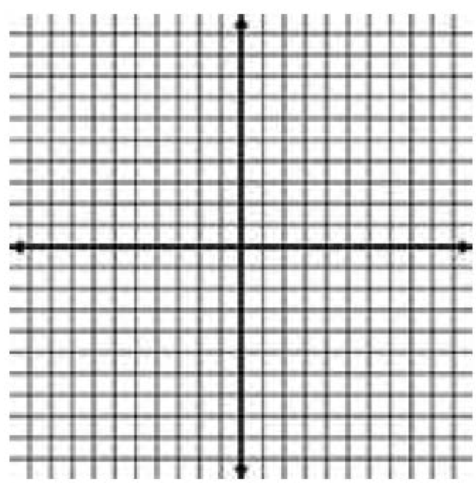


Domain: ____________ Domain: ____________

Range:______________ Range: _____________

b. $f(x)=\frac{1}{2}\sqrt{x}$

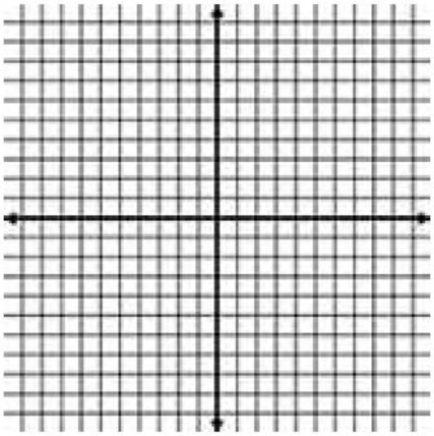


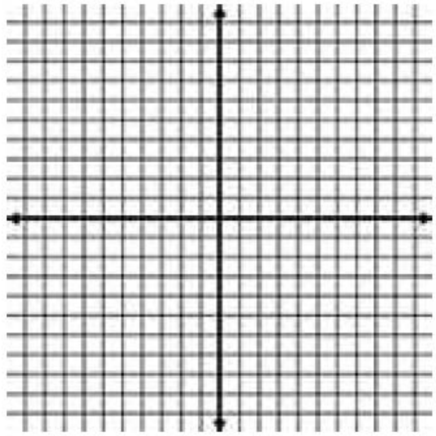


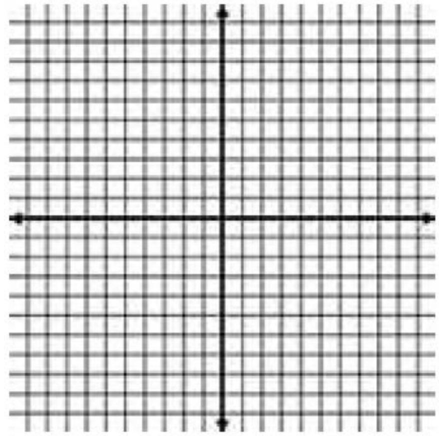
Domain: ____________ Domain: ____________

Range:______________ Range: _____________

c. $g(x)=\sqrt{-x}-2$



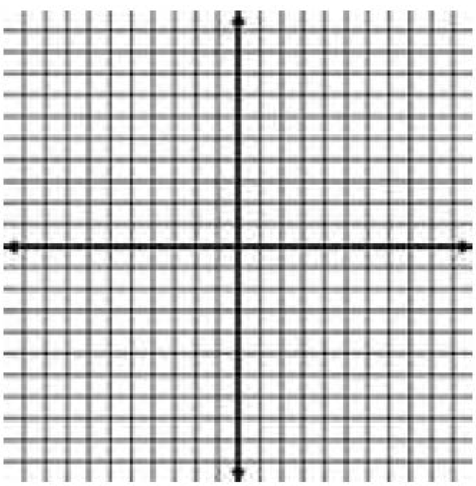


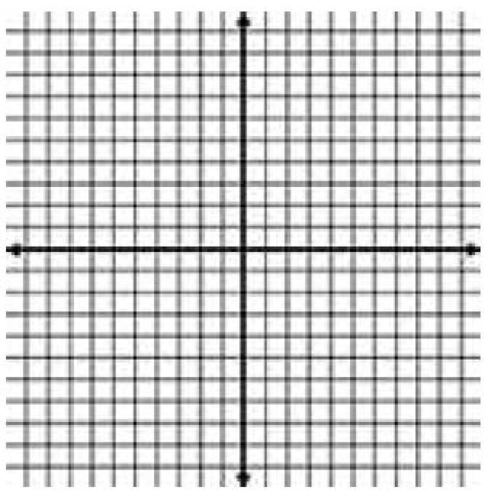


Domain: ____________ Domain: ____________ Domain: ____________

Range:_____________ Range: _____________ Range: _____________

d. $h(x)=\text{int}(-x)$





Domain: ____________ Domain: ____________

Range:______________ Range: _____________

Example 3: Suppose that the function $y=f(x)$ is decreasing on the interval $(-2,7)$.

a. Over what interval is the graph of $y=f(x+2)$ decreasing?

b. Over what interval is the graph of $y=f(x-5)$ decreasing?

c. What can be said about the graph of $y=-f(x)$?

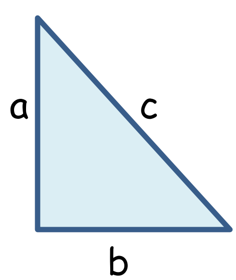
2.6: MATHEMATICAL MODELS: BUILDING FUNCTIONS

When you are done with your homework you should be able to…

- π Build and Analyze Functions

WARM-UP: Complete the following statements.

1. The sum of angles in a triangle is _______________.
2. The distance between the ordered pairs (x_1, y_1) and (x_2, y_2) is ____________________________________.
3. Distance = ____________________________.
4. The area of a rectangle is __________________________.
5. Perimeter is the ___________ of the ____________ of the ___________ of a polygon.
6. The area of a circle is ___________________.
7. The Pythagorean Theorem states: ________________________.



8. The volume of a right circular cylinder is ____________________.
9. The volume of a right circular cone is _______________________.
10. The volume of a sphere is _______________________.
11. The volume of a right rectangular prism is ___________________.
12. The volume of a right rectangular pyramid is _________________.

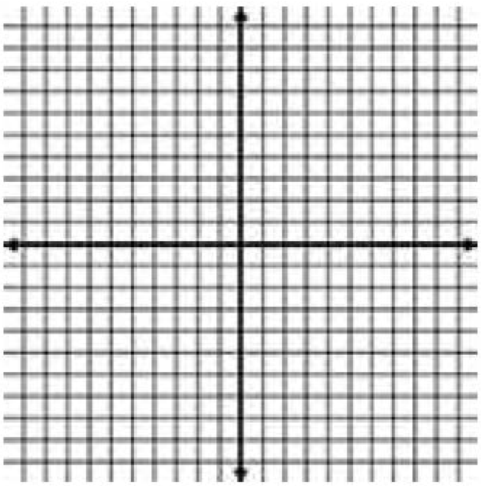
Example 1: Let $P=(x,y)$ be a point on the graph of $y=\frac{1}{x}$.

a. Express the distance d from P to the origin as a function of x.

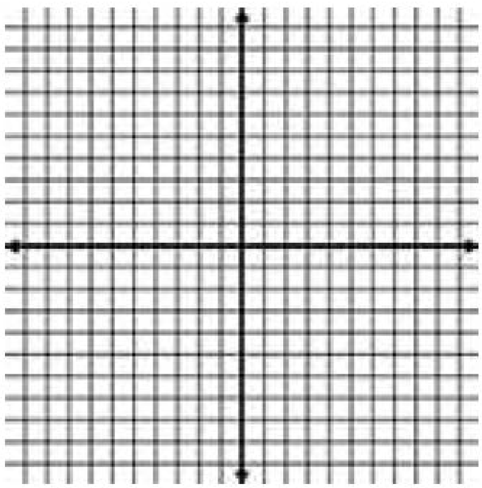
b. Use a graphing utility to graph $d=d(x)$.

c. For what values of x is d smallest?

Example 2: A right triangle has one vertex on the graph of $y = 9 - x^2$, $x > 0$, at (x, y), another at the origin, and the third on the positive *x*-axis at $(x, 0)$. Express the area A of the triangle as a function of x.



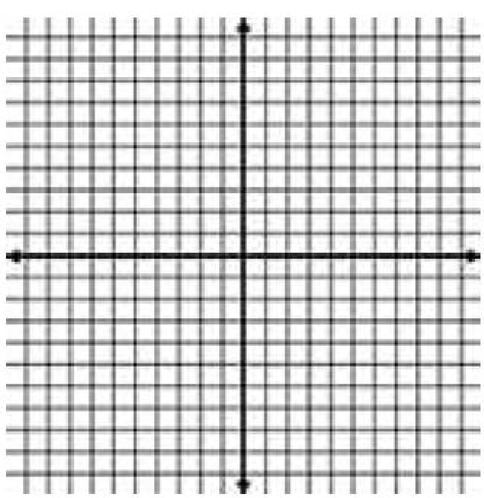
Example 3: A rectangle is inscribed in a semicircle of radius 2. Let $P=(x,y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle.



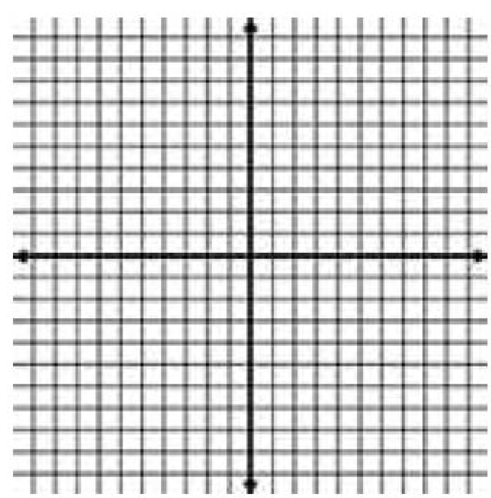
a. Express the area A of the rectangle as a function of x.

b. Express the perimeter p of the rectangle as a function of x.

c. Graph $A=A(x)$. For what value of x is A largest?



d. Graph $p=p(x)$. For what value of x is p largest?



Example 3: Two cars leave an intersection at the same time. One is headed south at a constant speed of 30 mph and the other is headed west at a constant speed of 40 mph. Build a model that expresses the distance d between the cars as a function of time t.

Example 4: An open box with a square base is required to have a volume of 10 cubic feet.

a. Express the amount A of material used to make such a box as a function of the length x of a side of the square base.

b. How much material is required for a base 1 foot by 1 foot?

c. How much material is required for a base 2 feet by 2 feet?

d. Use a graphing utility to graph $A = A(x)$. For what value of x is A smallest?

3.1: LINEAR FUNCTIONS AND THEIR PROPERTIES

When you are done with your homework you should be able to…

- π Graph Linear Functions
- π Use Average Rate of Change to Identify Linear functions
- π Determine Whether a Linear Function is Increasing, Decreasing, or Constant
- π Build Linear Models From Verbal Descriptions

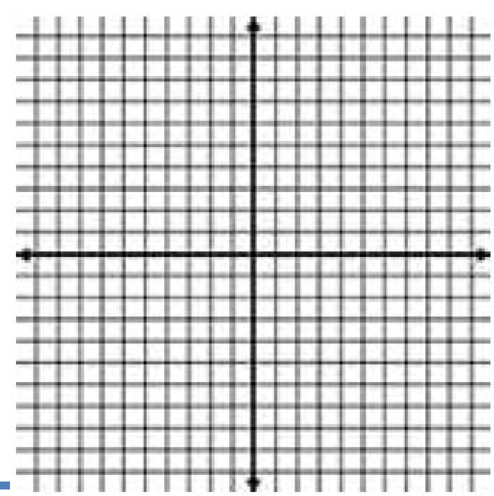
WARM-UP: Write the equation of the line which passes through the points $(-3,2)$ and $(5,7)$.

LINEAR FUNCTION

A linear function is a function of the form

The graph of a linear function is a _______________ with slope _____ and y-intercept ______. Its domain is the set of all ___________ numbers.

Example 1: Graph the linear function: $f(x) = -\frac{2}{3}x - 4$



AVERAGE RATE OF CHANGE OF A LINEAR FUNCTION

Linear functions have a ___________________ average rate of change. The average rate of change of ______________________________ is

PROOF:

Example 2: Determine whether the given function is linear or nonlinear. If it is linear, determine the equation of the line.

a.

x	$y = f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

b.

x	$y = f(x)$
-4	8
-2	4
0	0
2	-4
4	-8

INCREASING, DECREASING, AND CONSTANT LINEAR FUNCTIONS

A linear function ________________________ is…

____________________ over its domain if its ______________, ________, is ________________________.

____________________ over its domain if its ______________, ________, is ________________________.

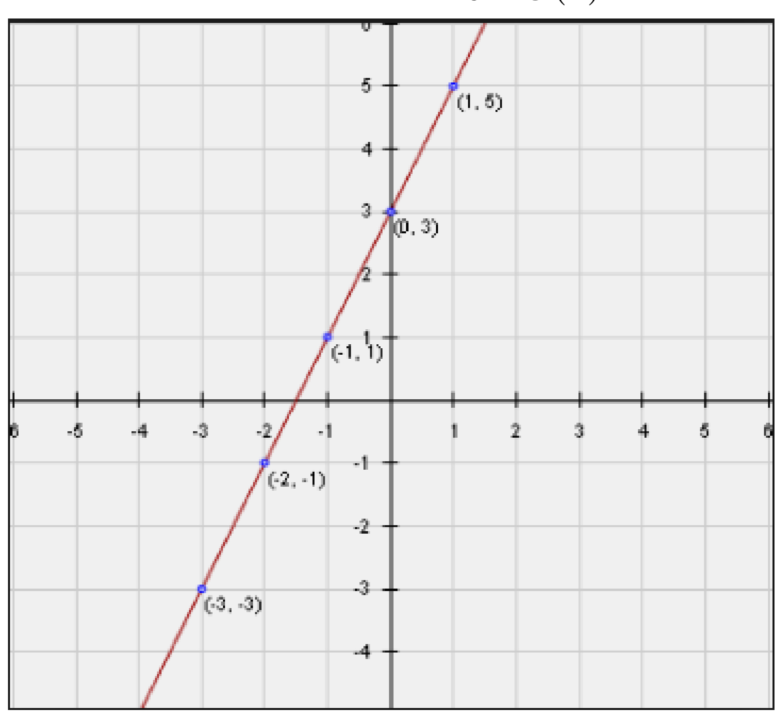
____________________ over its domain if its ______________, ________, is __________________.

Example 3: Determine whether the following linear functions are increasing, decreasing, or constant.

a. $f(x) = 2 - 4x$ b. $h(z) = -6$ c. $g(t) = 0.02t - 0.35$

Example 4: Consider the following linear function.

$y = g(x)$



a. Solve $g(x) = -1$

b. Solve $g(x) = 0$

c. Solve $g(x) \leq 3$

d. Solve $g(x) = 5$

e. Solve $g(x) > -1$

f. Solve $0 < g(x) < 5$

APPLICATIONS

1. The monthly cost C, in dollars, for international calls on a certain cellular phone plan is modeled by the function $C(x)=0.38x+5$, where x is the number of minutes used.
   a. What is the cost if you talk on the phone for $x=50$ minutes?

   b. Suppose that your monthly bill is $29.32. How many minutes did use the phone?

   c. Suppose that you budget yourself $60 per month for the phone. What is the maximum number of minutes that you can talk?

   d. What is the implied domain of C if there are 30 days in the month?

   e. Interpret the slope.

   f. Interpret the *y*-intercept.

2. Suppose that the quantity supplied S and quantity demanded D of hot dogs at a baseball game are given by the following functions:

$$S(p) = -2000 + 3000p$$
$$D(p) = 10,000 - 1000p$$

where p is the price of a hot dog.

a. Find the equilibrium price for hot dogs at the baseball game. What is the equilibrium quantity?

b. Determine the prices for which quantity demanded is less than quantity supplied.

c. What do you think will eventually happen to the price of hot dogs if quantity demanded is less than quantity supplied?

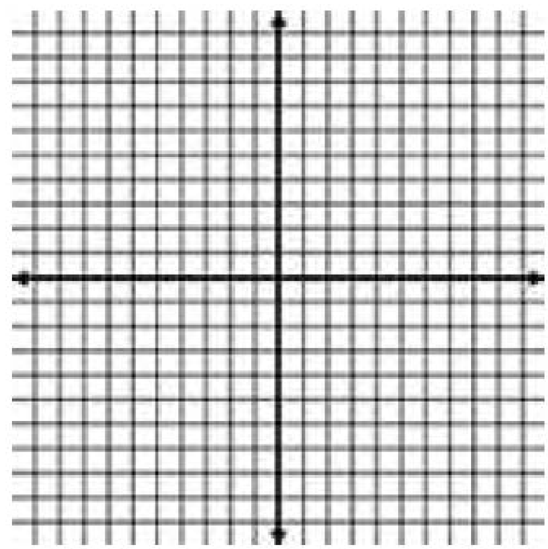
3.3: QUADRATIC FUNCTIONS AND THEIR PROPERTIES

When you are done with your homework, you should be able to…

- π Graph a Quadratic Function Using Transformations
- π Identify the Vertex and Axis of Symmetry of a Quadratic Function
- π Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts
- π Find a Quadratic Function Given Its Vertex and One Other Point
- π Find the Maximum or Minimum Value of a Quadratic Function

WARM-UP:

1. Graph $f(x) = x^2$.



2. Complete the square of the expression $x^2 + 6x - 1$

Graphs like the one we just did in the warm-up problem are the graphs of

______________________ functions, commonly called ____________________.

Parabolas open upward if the coefficient to the squared term is ______________

and downward if the coefficient to the squared term is ____________________.

Parabolas have a "fold" line, that is, they have ____________________ symmetry

about a vertical line. This vertical line is found when you find the ordered pair

where the ______________________ or _____________________ is located. This

ordered pair is called the ______________________ of the quadratic function.

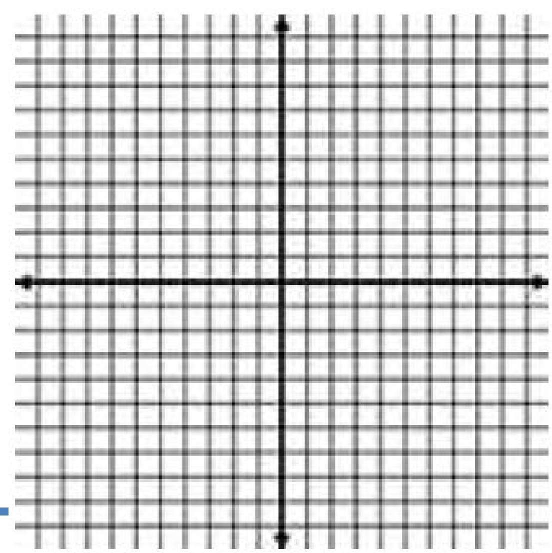
QUADRATIC FUNCTION

A quadratic function is a function of the form

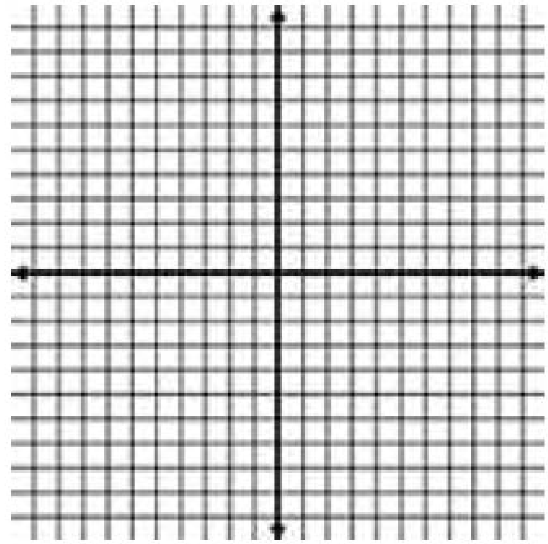
where *a*, *b*, and *c* are real numbers and ______________. The domain of a quadratic function is the set of ___________________ numbers.

Example 1: Graph using transformations.

a. $f(x) = 2x^2 + 4$.



b. $f(x) = -2x^2 + 6x + 2$



Now consider any quadratic function $f(x) = ax^2 + bx + c$.

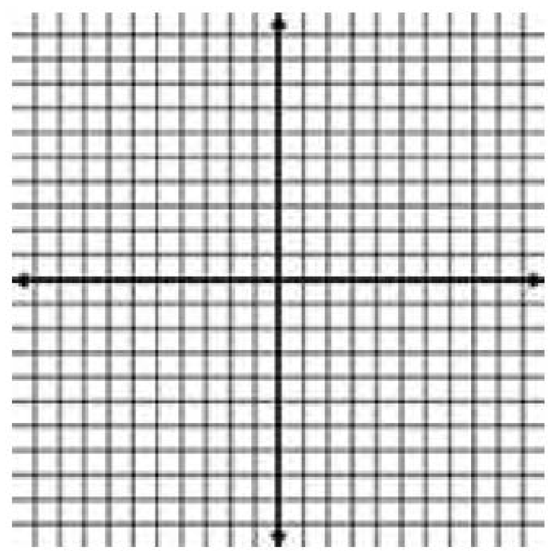
Based on these results, we conclude…

If ______________, and _____________________, then

where the vertex is the ordered pair _______________. If ___________, the _________________ occurs at the vertex and if ____________________ the __________________ occurs at the vertex.

Example 2: Consider the function $f(x) = -(x-3)^2 + 6$.

a. What is the vertex?



b. What is the axis of symmetry?

c. Find the x-intercept(s).

d. Find the y-intercept.

e. Sketch the graph.

Oftentimes, we are given quadratic equations in the form __________________.

When this happens, it is easier to use the fact the ________________ and find

_____ by evaluating _______________.

PROPERTIES OF THE GRAPH OF A QUADRATIC FUNCTION

$$f(x) = ax^2 + bx + c$$

Vertex: ____________________

Axis of Symmetry: _________________

If the parabola opens upward, ____________ and the vertex is a

________________ point.

If the parabola opens downward, ____________ and the vertex is a

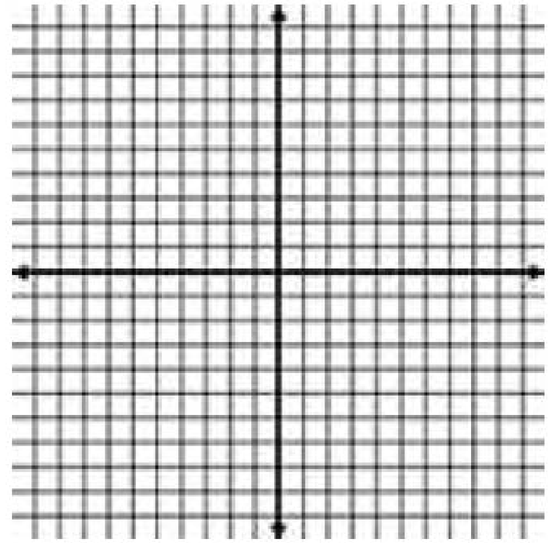
________________ point.

Example 3: Find the coordinates of the vertex for the parabola defined by the given quadratic function.

a. $f(x) = 3x^2 - 12x + 1$ b. $f(x) = -2x^2 + 7x - 4$ c. $f(x) = -3(x-2)^2 + 12$

Example 4: Consider the function $f(x) = 3x^2 - 8x + 2$.

a. What is the vertex?



b. What is the axis of symmetry?

c. Find the *x*-intercept(s).

d. Find the *y*-intercept.

e. Sketch the graph.

Example 5: The graph of the function $f(x) = ax^2 + bx + c$ has vertex at $(1, 4)$ and passes through the point $(-1, -8)$. Find *a*, *b*, and *c*.

STEPS FOR GRAPHING A QUADRATIC FUNCTION $f(x) = ax^2 + bx + c$, $a \neq 0$

Option 1

1. Complete the square in *x* to write the equation in the form ____________________________.
2. Graph the function in stages using ________________________________.

Option 2

1. Determine whether the parabola opens up (______________) or down (____________).
2. Determine the vertex: ____________________________
3. Determine the axis of symmetry ________________________.
4. Find the _______________________, if any.
   a. If ________________________, the graph of the quadratic function has _____________ _________________________.
   b. If _______________________, the ________________ is the _______________________.
   c. If ______________________, there are __________ ____________________.
5. Determine an additional point using ________________________.
6. Plot the points and sketch the graph.

APPLICATIONS

1. Find the point on the line $y = x + 1$ that is closest to point $(4,1)$.

2. The John Deere Company has found that the revenue, in dollars, from sales of riding mowers is a function of the unit price *p*, in dollars, that it charges. If the revenue *R* is $R(p) = -\frac{1}{2}p^2 + 1900p$ what unit price *p* should be charged to maximize revenue? What is the maximum revenue?

3.4: BUILD QUADRATIC MODELS FROM VERBAL DESCRIPTIONS

When you are done with your homework, you should be able to…

- π Build Quadratic Models From Verbal Descriptions

WARM-UP: Find the vertex of the quadratic function $f(x) = -2x^2 - x + 5$.

Example 1: The price *p* (in dollars) and the quantity *x* sold of a certain product obey the demand equation $p = -\frac{1}{3}x + 100$.

a. Find a model that expresses the revenue *R* as a function of *x*.

b. What is the domain of *R*?

c. What is the revenue if 100 units are sold?

d. What quantity *x* maximizes revenue? What is the maximum revenue?

e. What price should the company charge to maximize revenue?

Example 2: A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

Example 3: A parabolic arch has a span of 120 feet and a maximum height of 25 feet. Choose suitable rectangular axes and find an equation of the parabola. Then calculate the height of the arch at points 10 feet, 20 feet, and 40 feet from the center.

Example 4: A projectile is fired at an inclination of 45° to the horizontal, with a muzzle velocity of 100 feet per second. The height *h* of the projectile is modeled by $h(x)=\frac{-32x^2}{(100)^2}+x$ where *x* is the horizontal distance of the projectile from the firing point.

a. At what horizontal distance from the firing point is the height of the projectile a maximum?

b. Find the maximum height of the projectile.

c. At what horizontal distance from the firing point will the projectile strike the ground?

d. Using a graphing calculator, graph the function *h*, $0 \le x \le 350$.

e. Use a graphing calculator to verify the results obtained in parts b and c.

f. When the height of the projectile is 50 feet above the ground, how far has it traveled horizontally?

3.5: INEQUALITIES INVOLVING QUADRATIC FUNCTIONS

When you are done with your homework, you should be able to…

- π Solve Inequalities Involving a Quadratic Function

WARM-UP: Find the zeroes of $f(x)=3x^2-x-5$.

STEPS TO SOLVE A QUADRATIC INEQUALITY

1. Find the ________________ of the quadratic function_______________.
2. Draw a number line, using the ____________ to separate the number line into intervals.
3. Choose a number from each interval and evaluate the number in ________________________.
   a. If you get a positive result, that interval is the solution for inequalities with ________ or ________.
   b. If you get a negative result, that interval is the solution for inequalities with __________ or __________.
4. Write your result in set and interval notation. If you have an "or equals to" situation, the _________________ are included as long as _______ or ________ is not in the interval. If you have more than one interval that satisfies the inequality, use the word "or" in between the inequalities in set-builder notation or use the $\cup$ symbol to join the intervals in interval notation.

Example 1: Solve each inequality. Verify your results using a graphing calculator.

a. $x^2+3x-10>0$

b. $6x^2 \leq 6+5x$

c. $2\left(2x^2-3x\right)>-9$

4.1: POLYNOMIAL FUNCTIONS AND MODELS

When you are done with your homework, you should be able to…

- π Identify Polynomial Functions and Their Degree
- π Graph Polynomial Functions Using Transformations
- π Identify the Real Zeros of a Polynomial Function and Their Multiplicity
- π Analyze the Graph of a Polynomial Function

WARM-UP: Use your graphing calculator to graph…

a. $f\left(x\right)=x^2$

b. $f\left(x\right)=-x^2$

c. $f\left(x\right)=x^3$

d. $f\left(x\right)=-x^3$

POLYNOMIAL FUNCTION

A polynomial function is a function of the form

Where $a_n, a_{n-1}, \ldots, a_1, a_0$ are ____________ numbers and *n* is a ____________________ ___________________. The domain of a polynomial function is the set of ____________________ numbers. The _______________ of a polynomial function is the _________________ ________________ of ______ that appears. The ___________ polynomial function, _________________________ is not assigned a degree.

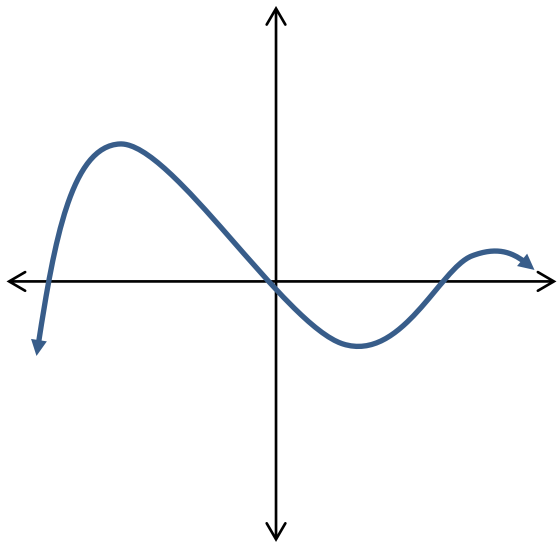
Example 1: Determine which of the following are polynomial functions. For those that are, state the degree. For those that are not, state why not.

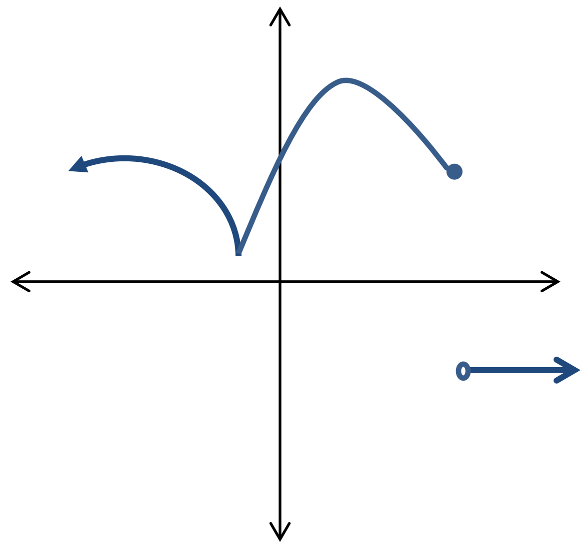
a. $f(x) = 4x^2 - x^{2/3} + 1$

b. $g(x) = -x^{10} + \frac{3}{4}x^4 + x$

c. $f(x) = 5x^3 - x^{-2} + 10$

One objective of this section is to _______________ the graph of a polynomial function. The graph of a polynomial function is both _____________ and ___________________. A _______________ graph has no _______________ corners or _______________. A ____________________ graph has no gaps of holes and can be drawn without lifting pencil from paper.





POWER FUNCTIONS

A power function of degree *n* is a ________________ function of the form

Where a is a real number, __________, and __________ is an integer.

Example 2: Give three examples of power functions.

a.

b.

c.

Example 3: Graph $f(x)=x^2$, $f(x)=x^4$ and $f(x)=x^{10}$ in the same window on your graphing calculator.

What do you notice about the end behavior of these graphs?

What are the x-intercept(s)?

PROPERTIES OF POWER FUNCTIONS, $f(x)=x^n$, *n* IS AN EVEN INTEGER

1. f is an ____________ function, so its graph is symmetric with respect to the ______________________.
2. The domain is the set of all ______________ numbers. The range is the set of all _____________________ ____________ numbers.
3. The graph always contains the points ________, ________, and ________.
4. As the exponent *n* increases in magnitude, the graph increases more rapidly when __________________; but for *x* near the origin, the graph tends to ________________ out and lie ______________ to the ______________.

Example 4: Graph $f(x)=x^3$, $f(x)=x^5$ and $f(x)=x^{11}$ in the same window on your graphing calculator.

What do you notice about the end behavior of these graphs?

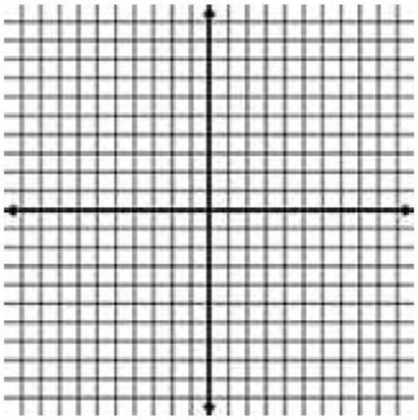
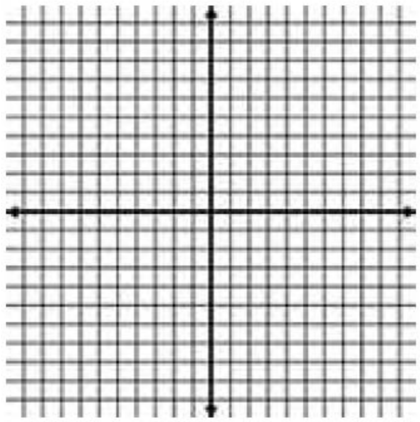
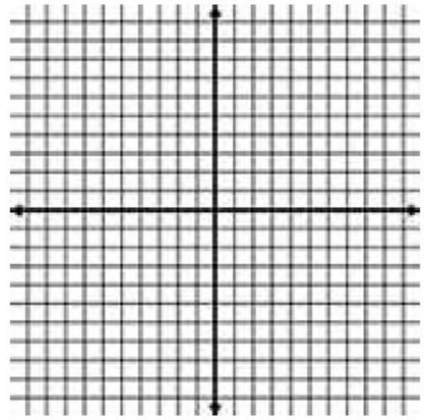
What are the x-intercept(s)?

PROPERTIES OF POWER FUNCTIONS, $f(x)=x^n$, *n* IS AN ODD INTEGER

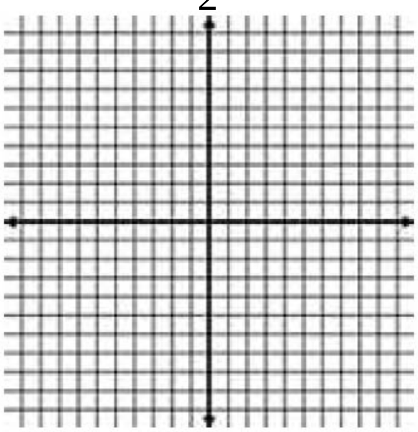
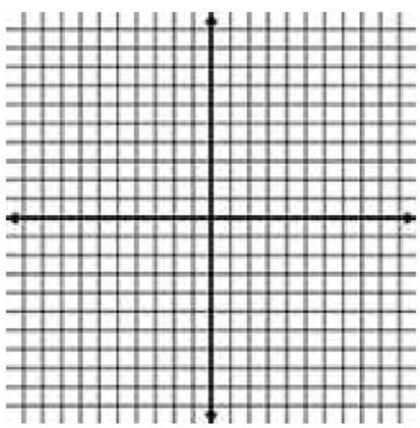
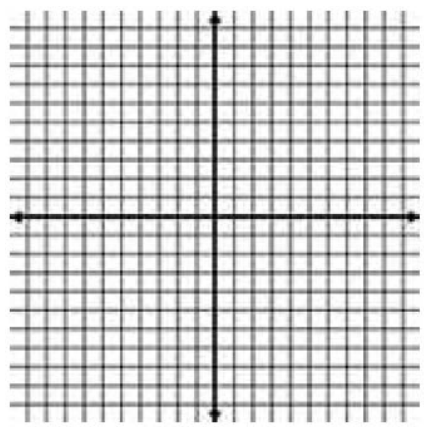
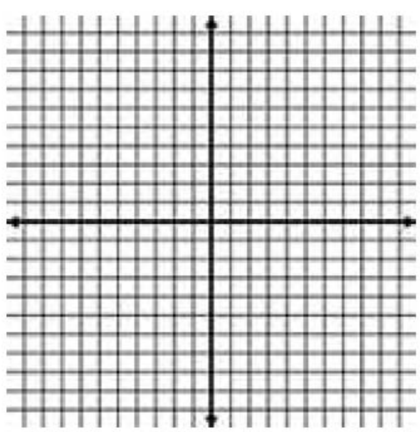
1. f is an ____________ function, so its graph is symmetric with respect to the ______________________.
2. The domain is the set of all ______________ numbers. The range is the set of all ____________ numbers.
3. The graph always contains the points ________, ________, and ________.
4. As the exponent *n* increases in magnitude, the graph increases more rapidly when __________________; but for *x* near the origin, the graph tends to ________________ out and lie ______________ to the ______________.

Example 5: Graph by hand using transformations.

a. $f(x) = (2 - x)^4$

b. $g(x) = \frac{1}{2}(x - 1)^5 - 2$

REAL ZEROS

If f is a function and *r* is a real number for which $f(r) = 0$, then *r* is called a

______________ _____________ of ______.

As a consequence of this definition, the following statements are equivalent:

1. *r* is a real zero of a polynomial function f.
2. *r* is an _________________________ of the graph of f.
3. $x - r$ is a __________________ of f.
4. *r* is a solution to the equation __________________.

Example 6: Form a polynomial function whose real zeros are -3, -1, 2, and 5 has degree 4.

REPEATED ZEROS

If $(x-r)^m$ is a factor of a polynomial f and $(x-r)^{m+1}$ is not a factor of f, then r is called a ___________ of ______________________ _______ of f.

TURNING POINTS

If f is a polynomial of degree *n*, then f has at most _______________ turning points.

If the graph of a polynomial function f has *n* – 1 turning points, the degree of f is at least _________.

END BEHAVIOR

For ________________ values of *x*, either positive or negative, the graph of the polynomial function

resembles the graph of the power function

Example 7: Consider the function $f(x)=\left(x+\sqrt{3}\right)^2(x-2)^4$.

a. List each real zero and its multiplicity.

b. Determine whether the graph crosses or touches the *x*-axis at each *x*-intercept.

c. Determine the maximum number of turning points on the graph.

d. Determine the end behavior.

SUMMARY: GRAPH OF A POLYNOMIAL FUNCTION

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, $a \neq 0$

Degree of the polynomial function f : __________

Graph is _________________ and ______________________.

Maximum number of turning points: _________________

At a zero of even multiplicity: The graph of f ____________ the ____________.

At a zero of even multiplicity: The graph of f ____________ the ____________.

Between zeros, the graph of f is either above or below the _________________.

End Behavior: For large_________, the graph of f behaves like_______________.

SUMMARY: ANALYZING THE GRAPH OF A POLYNOMIAL FUNCTION

1. Determine the ___________ __________________ of the graph of the function.

2. Find the ___ and ___ intercepts of the graph of the function.

3. Determine the ________________ of the function and their ________________________. Use this information to determine whether the graph ________________ or ____________________ the *x*-axis.

4. Use a graphing calculator to graph the function.

5. Approximate the _________________ points of the graph.
6. Use the information in steps 1-5 to draw a complete graph of the function by hand.
7. Find the _________________ and __________________ of the function.

8. Use the graph to determine where the function is __________________ and where it is _____________________.

Example 8: Analyze $f(x)=x^2(x^2+1)(x+4)$.

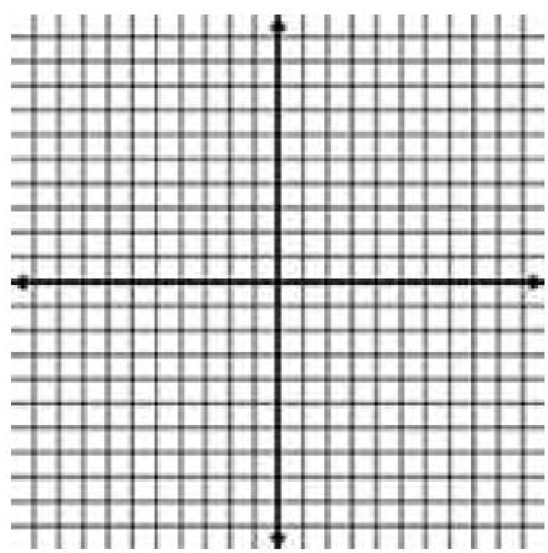
1. End Behavior.

2. Intercepts.

3. Multiplicity.

4. Use a graphing calculator to graph the function.

5. Turning points.
6. Use the information in steps 1-5 to draw a complete graph of the function by hand.



7. Domain and range.

8. Increasing/decreasing intervals.

4.2: THE REAL ZEROS OF A POLYNOMIAL FUNCTION

When you are done with your homework, you should be able to…

- π Use the Remainder and Factor Theorems
- π Use the Rational Zeros Theorem to List the Potential Rational Zeros of a Polynomial Function
- π Find the Real Zeros of a Polynomial Function
- π Solve Polynomial Equations
- π Use the Theorem for Bounds on Zeros
- π Use the Intermediate Value Theorem

WARM-UP: Divide.

$$\frac{x^2 - x + 1}{x - 1}$$

The numerator of the expression we just divided was the ________________ in the division problem. The denominator was the ________________. Our result had a ___________________ plus a ____________________ in *x*.

DIVISION ALGORITHM FOR POLYNOMIALS

If $f(x)$ and $g(x)$ denote polynomial functions and if $g(x)$ is a polynomial function whose degree is greater than zero, then there are ________________ polynomial functions $q(x)$ and $r(x)$ such that

where $r(x)$ is either the zero polynomial or a polynomial function of degree less than that of $g(x)$.

REMAINDER THEOREM

Let f be a polynomial function. If $f\left(x\right)$ is divided by $x-c$, then the remainder is

____________________.

Example 1: Find the remainder if $f\left(x\right)=2x^4-8x^3-10$ is divided by

a. $x-9$

b. $x+1$

FACTOR THEOREM

Let f be a polynomial function. Then $x-c$ is a factor of $f\left(x\right)$ **if and only if**

____________________.

1. If $f\left(c\right)=0$, then ______________ is a _______________ of $f\left(x\right)$.

2. If $x-c$ is a factor of $x-c$, then _________________.

Example 2: Use the Factor Theorem to determine whether the function $f(x) = 3x^3 - 6x^2 + 4x - 8$ has the factor

a. $x+1$

b. $x-2$

NUMBER OF REAL ZEROS

A polynomial function cannot have more real ________________ than its __________________.

RATIONAL ZEROS THEOREM

Let f be a polynomial function of degree 1 or higher of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, $a_n \neq 0$, $a_0 \neq 0$ where each coefficient is an ____________________. If ____________, in lowest terms, is a rational zero of f, then p must be a ______________ of a_0 and q must be a ______________ of a_n.

Example 3: Determine the maximum number of real zeros and list the potential rational zeros of $f(x) = -2x^4 + 12x^3 + x^2 - 24x - 10$.

SUMMARY: STEPS FOR FINDING THE REAL ZEROS OF A POLYNOMIAL FUNCTION

1. Use the degree of the polynomial function to determine the maximum number of real ___________________.

2. If the polynomial function has _________________ coefficients, use the _________________ Zeros Theorem to identify those rational numbers that potentially can be _________________.

3. Graph the polynomial function using your graphing calculator to find the best choice of potential rational zeros.

4. Use the ________________ Theorem to determine if the potential rational zero is a _______________.If it is, use synthetic division or long division to ______________ the polynomial function. Each time that a zero (and thus a _______________) is found, _______________ step 4 on the _________________ equation. In attempting to find the zeros, remember to use the factoring techniques that you already _________________!!!

Example 4: Solve the equation $f(x) = 6x^4 - x^2 + 2$.

Every polynomial function (with real coefficients) can be ____________________ ________________ into a _________________ of _________________ factors and/or ____________________ ________________________ factors.

A polynomial function (with real coefficients) of ___________ degree has at least __________ ______________ _________________.

BOUNDS ON ZEROS

Let f denote a polynomial function whose leading coefficient is 1.

$$f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

A bound M on the real zeros of f is the smaller of the two numbers

where ____________________ means "choose the largest entry in { }".

Example 5: Find a bound to the zeros of each polynomial function. Use the bounds to obtain a complete graph of f.

a. $f(x) = 3x^3 - 2x^2 + x + 4$

b. $f(x) = 4x^4 - 12x^3 + 27x^2 - 54x + 81$

INTERMEDIATE VALUE THEOREM

Let f denote a continuous function. If $a < b$ and if $f(a)$ and $f(b)$ are of ______________________ sign, then f has at least one zero between _____ and ______.

Example 6: Use the Intermediate Value Theorem to show that the function $f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + x + 2$ has a zero on the closed interval $[1.7, 1.8]$. Approximate the zero rounded to two decimal places.

Example 7: Find the real zeros of f. Use the real zeros to factor f.

a. $f(x) = x^3 + 8x^2 + 11x - 20$

b. $f(x) = 4x^4 + 15x^2 - 4$

Example 8: Find the real solutions of each equation.

a. $2x^3 - 11x^2 + 10x + 8 = 0$

b. $x^4 - 2x^3 + 10x^2 - 18x + 9 = 0$

APPLICATIONS

1. Find *k* such that $f(x) = x^4 - kx^3 + kx^2 + 1$ has the factor $x + 2$.

2. What is the length of the edge of a cube its volume could be doubled by an increase of 6 centimeters in one edge, an increase of 12 centimeters in a second edge, and a decrease of 4 centimeters in the third edge?

4.3: COMPLEX ZEROS: THE FUNDAMENTAL THEOREM OF ALGEBRA

When you are done with your homework, you should be able to…

- π Use the Conjugate Pairs Theorem
- π Find a Polynomial Function with Specified Zeros
- π Find the Complex Zeros of a Polynomial Function

WARM-UP: Find all complex zeros of $f(x) = 5x^2 - x + 2$.

COMPLEX POLYNOMIAL FUNCTIONS

A variable in the ____________________ number system is referred to as a complex variable. A ____________________ ______________________ function f of degree *n* is of the form

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are complex numbers, $a_n \neq 0$, *n* is a nonnegative integer, and *x* is a complex variable. As before, a_n is called the leading coefficient of f. A complex number *r* is called a complex zero of f if ___________________.

FUNDAMENTAL THEOREM OF ALGEBRA

Every _______________ polynomial function $f(x)$ of degree $n \geq 1$ has at least one

complex ___________________.

Every complex polynomial function $f(x)$ of degree $n \geq 1$ can be factored into *n* linear factors (not necessarily _________________) of the form

where $a_n, r_1, r_2, \ldots, r_n$ are complex numbers. That is, _______________ complex polynomial function of degree $n \geq 1$ has ___________________ *n* complex ____________________, some of which may __________________.

CONJUGATE PAIRS THEOREM

Let $f(x)$ be a polynomial function whose coefficients are real numbers. If

_______________________ is a zero of f, the complex conjugate

_______________________ is also a zero of f.

Example 1: Find a polynomial function f of degree 4 whose coefficients are real numbers and that has the zeros -2, 1, and $2-i$. Use your graphing calculator to verify your result.

Example 2: Find the complex zeros of each polynomial function. Write f in factored form.

a. $f(x) = x^4 - 1$

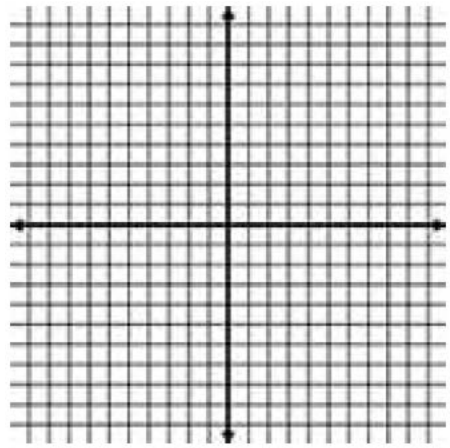
b. $f(x) = x^3 + 13x^2 + 57x + 85$

4.4: PROPERTIES OF RATIONAL FUNCTIONS

When you are done with your homework, you should be able to…

- π Find the Domain of a Rational Function
- π Find the Vertical Asymptotes of a Rational Function
- π Find the Horizontal or Oblique Asymptote of a Rational Function

WARM-UP: Graph $f(x)=\frac{1}{x}$.



What is the domain?

What is the range?

RATIONAL FUNCTIONS

A rational function is a function of the form

where p and q are polynomial functions and q is not the ______________ polynomial. The domain of a rational function is the set of all ____________ numbers except those for which the _________________ _____ is ________.

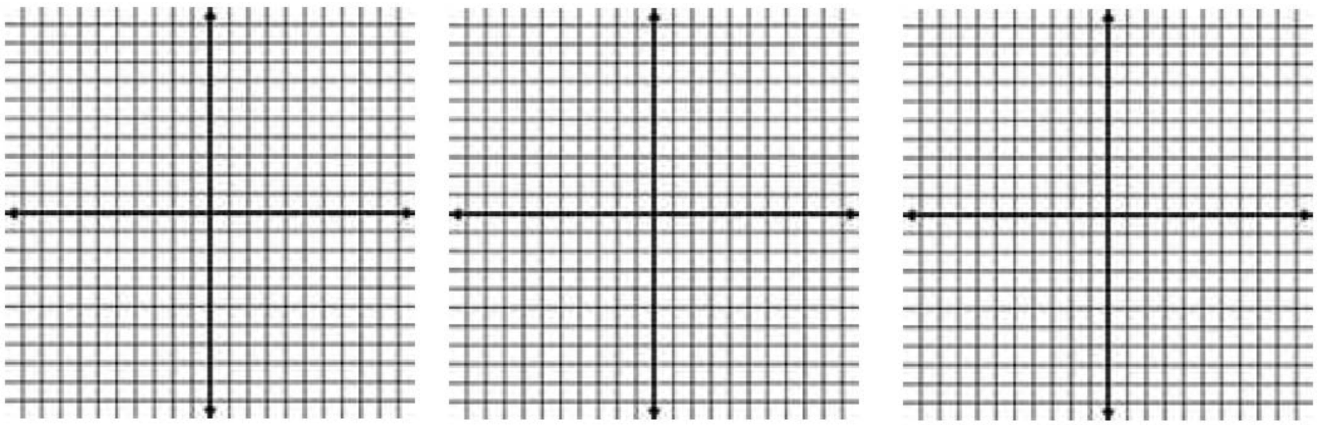
Example 1: Find the domain of the following rational functions.

a. $R(x)=\frac{5x^2}{3+x}$

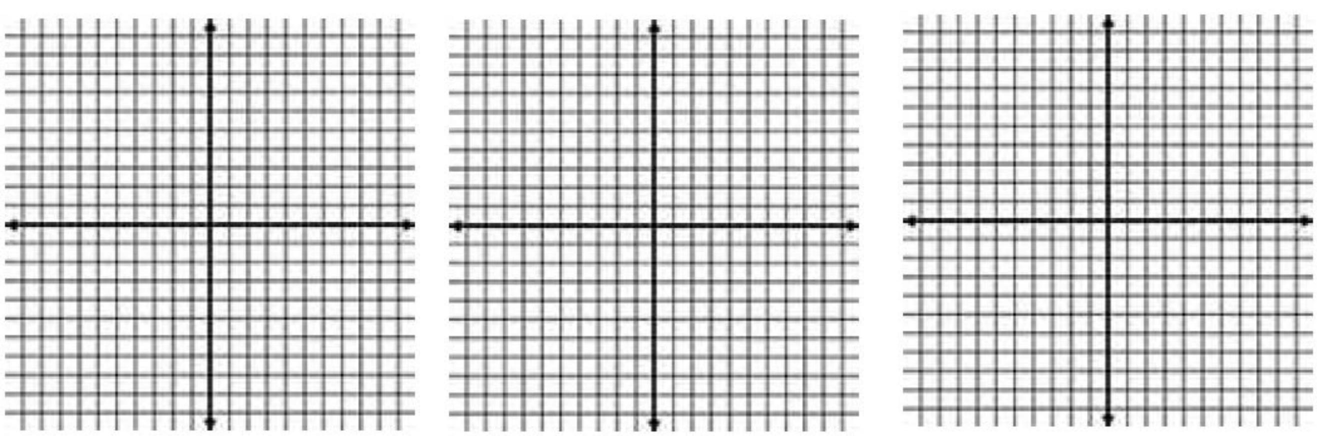
b. $F(x)=\frac{-x(1-x)}{3x^2+5x-2}$

Example 2: Graph the rational function using transformations.

a. $R(x)=\dfrac{1}{x-1}+1$



b. $G(x)=\dfrac{-2}{x^2-6x+9}$



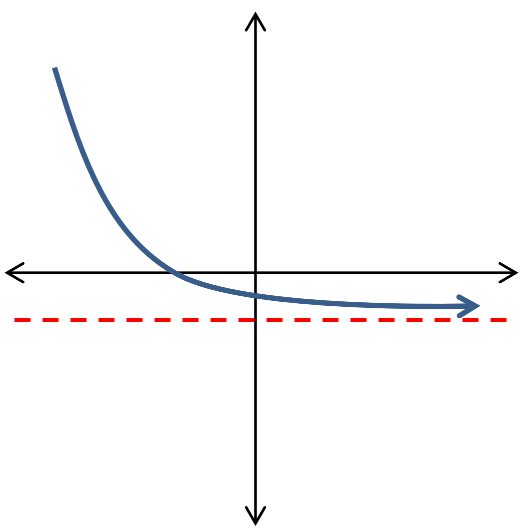
HORIZONTAL AND VERTICAL ASYMPTOTES

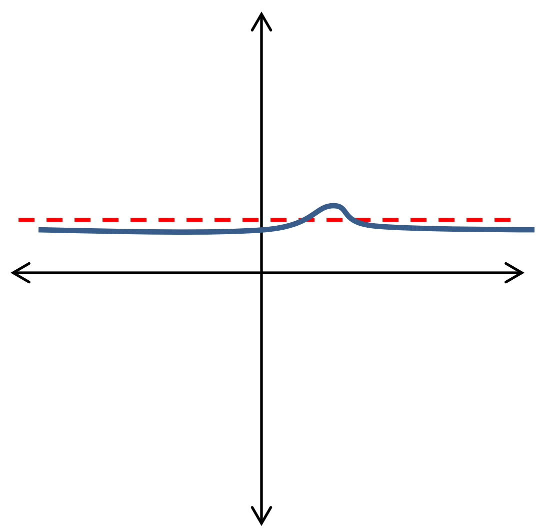
Let R denote a function:

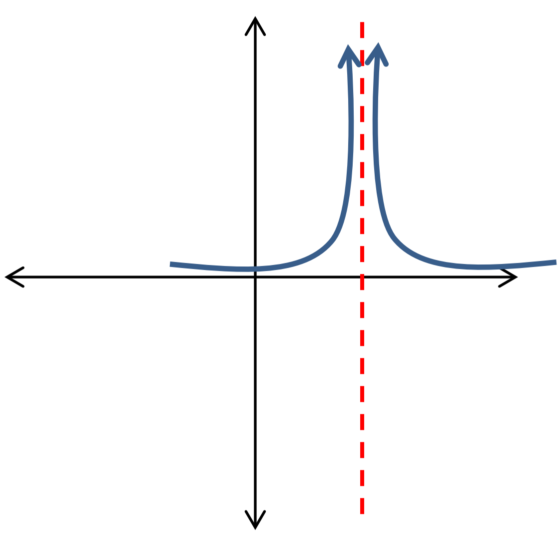
If, as ________________ or as ________________, the values of $R(x)$ approach some ________________ number ________, then the line ________________ is a ________________ asymptote of the graph of R.

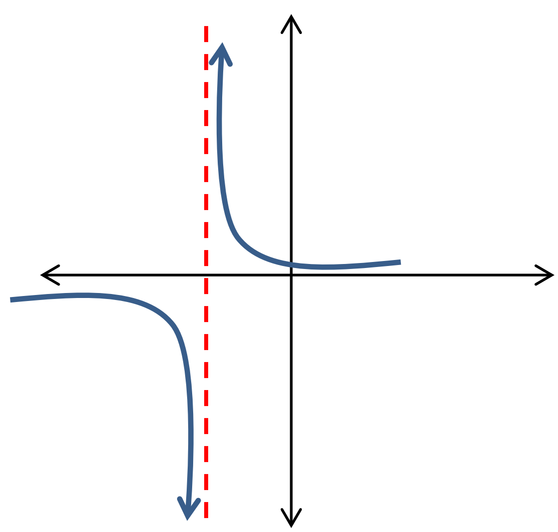
If, as x approaches some number c, the values ________________, then the line ________________ is a ______________________ asymptote of the graph of R.

The graph of R ________________ intersects a vertical asymptote!!!









LOCATING VERTICAL ASYMPTOTES

A rational function $R(x)=\dfrac{p(x)}{q(x)}$ in ________________ terms, will have a vertical asymptote _______________ if r is a real zero of the __________________ _____.

Example 3: Find the vertical asymptotes, if any, of the graph of each rational function.

a. $F(x)=\dfrac{x-1}{x^2+4}$

b. $Q(x)=\dfrac{x}{x^2+12x+32}$

c. $G(x)=\dfrac{x+10}{x^2-100}$

HORIZONTAL AND OBLIQUE ASYMPTOTES

To find horizontal and oblique asymptotes, we need to check out the ________ behavior of the function. If a rational function $R(x)$ is _______________, that is, the degree of the numerator is ___________ than the degree of the denominator, then as _________________, or as __________________ the value of __________ approaches _________. It follows that the line ______________ is a _____________________ asymptote of the graph. If a rational function is

_____________________, that is, the degree of the numerator is _____________ than or ___________ to the denominator, we write the rational function as the sum of a polynomial function $f(x)$ plus a proper rational function $\frac{r(x)}{q(x)}$ using long division. That is _________________________________, where $f(x)$ is a polynomial function and $\frac{r(x)}{q(x)}$ is a proper rational function. Since $\frac{r(x)}{q(x)}$ is proper, then ________________ as ______________ or as ________________. As a result,

So we have three possibilities:

1. If ___________________, a constant, then the line ________________ is a

   ____________________ asymptote of the graph of R.

2. If ______________, _________________, then the line _____________ is

   an ______________________ asymptote of the graph of R.

3. In all other cases, the graph of ___________ approaches the graph of

   _____, and there are no horizontal or oblique asymptotes.

Example 4: Find the vertical, horizontal, and oblique asymptotes, if any, of each rational function.

a. $R(x)=\dfrac{7x-8}{6x+1}$

b. $F(x)=\dfrac{x^3+4}{2x^2-x+6}$

c. $G(x)=\dfrac{4x}{9x^2-121}$

4.5: THE GRAPH OF A RATIONAL FUNCTION

When you are done with your homework, you should be able to…

- π Analyze the Graph of a Rational Function
- π Solve Applied Problems Involving Rational Functions

WARM-UP: Find the zeros of $R(x)=\frac{5x^2+3x-4}{x^2+1}$. Give both the exact result and then round to the nearest tenth.

SUMMARY: ANALYZING THE GRAPH OF A RATIONAL FUNCTION

1. _______________ the numerator and denominator of *R*. Find the __________ of the rational function.

2. Write *R* in ___________________ terms.

3. Locate the intercepts of the graph. The ___________________, if any, of $R(x)=\frac{p(x)}{q(x)}$ in lowest terms satisfy the equation ______________. The ________________, if there is one, is _____________.

4. Locate the _______________________. of the function. The ___________________, if any, of $R(x)=\frac{p(x)}{q(x)}$ in lowest terms are found by identifying the ______________ _________________ of ___________.

   Each _______________ of the ___________________ gives rise to a _____________________ asymptote.

5. Locate the _____________________ or ______________________ asymptote, if one exists. Determine _________________, if any, at which the graph of *R* intersects this asymptote. (See Section 4.4, if you forgot☺)

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.

Example 1: Analyze $R(x)=\frac{2x+4}{x-1}$. If the rational function does not have a characteristic listed below, write "none".

1. Factor and identify the domain of *R*.

2. Write *R* in lowest terms.

3. Find the
   a. *x*-intercept(s)

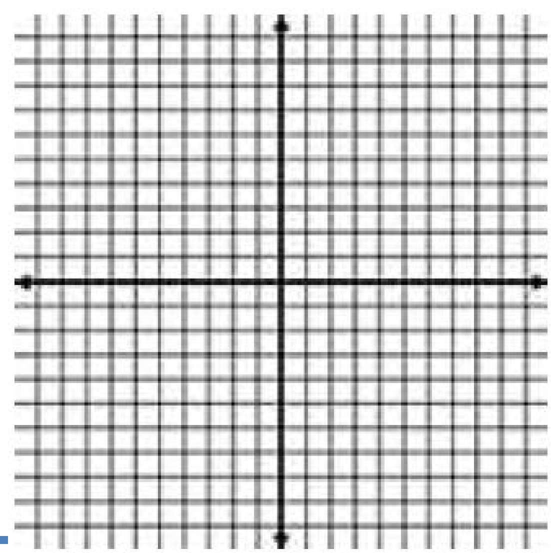
   b. *y*-intercept

4. Locate the vertical asymptote(s).

5. Locate the horizontal or oblique asymptote.

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.



Example 2: Analyze $R(x)=\dfrac{x^2+3x-10}{x^2+8x+15}$. If the rational function does not have a characteristic listed below, write "none".

1. Factor and identify the domain of *R*.

2. Write *R* in lowest terms.

3. Find the
   a. *x*-intercept(s)

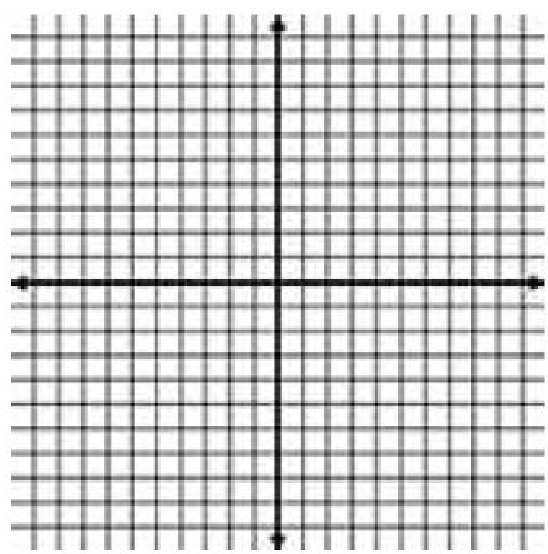
   b. *y*-intercept

4. Locate the vertical asymptote(s).

5. Locate the horizontal or oblique asymptote.

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.



Example 3: Analyze $R(x) = 2x + \frac{9}{x}$. If the rational function does not have a characteristic listed below, write "none".

1. Factor and identify the domain of *R*.

2. Write *R* in lowest terms.

3. Find the
   a. *x*-intercept(s)

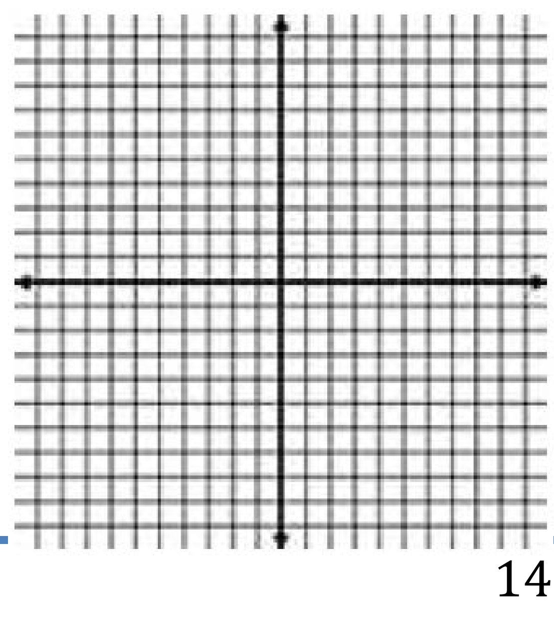
   b. *y*-intercept

4. Locate the vertical asymptote(s).

5. Locate the horizontal or oblique asymptote.

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.



APPLICATION

1. The concentration *C* of a certain drug in a patient's bloodstream t minutes after injection is given by $C(t)=\frac{50t}{t^2+25}$.

   a. Find the horizontal asymptote of $C(t)$. What happens to the concentration of the drug as t increases?

   b. Using your graphing calculator, graph $C=C(t)$.

   c. Determine the time at which the concentration is highest.

4.6: POLYNOMIAL AND RATIONAL INEQUALITIES

When you are done with your homework, you should be able to…

- π Solve Polynomial Inequalities Algebraically and Graphically
- π Solve Rational Inequalities Algebraically and Graphically

WARM-UP: Find the domain of $R(x)=\dfrac{x^2-x-3}{4x^2-1}$.

STEPS FOR SOLVING POLYNOMIAL AND RATIONAL INEQUALITIES

1. Write the inequality so that a polynomial or rational expression f is on the left side and ______________ is on the right side in one of the following forms:

   For rational expressions, be sure that the left side is written as a ________________ quotient AND find the ________________ of f.

2. Determine the real numbers at which the expression f equals _________ and, if the expression is rational, the real numbers at which the expression f is ____________________.

3. Use the numbers found in step 2 to separate the real _________________ line into _____________________.

4. Select a number in each ___________________ and evaluate f at that number.
   a. If the value of f is ________________, then __________ for all numbers ______ in the interval.
   b. If the value of f is ________________, then __________ for all numbers ______ in the interval.

   If the inequality is not strict (_______ or _______), include the solutions of ______________ that are in the ______________ of ______ in the solution set. Be sure to ________________ values of _____ where _____ is __________________________.

Example 1: Solve each inequality algebraically. Verify your results using a graphing calculator.

a. $(x-5)(x+2)^2 > 0$

b. $x^4 < 9x^2$

c. $\frac{5}{x-3} < \frac{3}{x+1}$

d. $\dfrac{x\left(x^2+1\right)\left(x-2\right)}{\left(x-1\right)\left(x+1\right)} \geq 0$

5.1: COMPOSITE FUNCTIONS

When you are done with your homework, you should be able to…

- π Form a Composite Function
- π Find the Domain of a Composite Function

WARM-UP: Consider the function $f(x)=\sqrt{x}$.

a. What is the domain of f?

b. Evaluate

  i. $f(16)$

  ii. $f(a)$

  iii. $f(5x)$

What must be true about *x* in this part for us to evaluate f?

COMPOSITE FUNCTIONS

Given two functions f and g, the ________________ function, denoted by $f \circ g$ (read as f ________________ with g) is defined by

The domain of ___________ is the set of all numbers __________ in the domain of __________ such that ____________ is in the domain of f.

Example 1: Let $f(x) = -x^2 + 3$ and $g(x) = 1 - x$. Find

a. $(f \circ g)(1)$

b. $(g \circ f)(1)$

c. $(f \circ f)(-2)$

d. $(g \circ g)(-1)$

Example 2: Let $f(x) = -x^2 + 3$ and $g(x) = 1 - x$. Find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

d. $(g \circ g)(x)$

e. $\dfrac{f(x+h) - f(x)}{h}$

Example 3: Let $f(x)=\sqrt{x-1}$ and $g(x)=x^3$. What is the domain of $f \circ g$?

Example 4: Let $f(x)=\dfrac{5}{2x-7}$ and $g(x)=x+2$. What is the domain of $f \circ g$?

Example 5: Let $f(x)=x^5$ and $g(x)=\sqrt[5]{x}$. Find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

What did you notice? What do you think this means?

Example 6: Find functions f and g so that $H(x) = \left(1 + x^2\right)^6$

Example 7: Find functions f and g so that $H(x) = |5x - 8|$

APPLICATION

The spread of oil leaking from a tanker is in the shape of a circle. If the radius *r* (in feet) of the spread after *t* hours is $r(t) = 200\sqrt{t}$, find the area *A* of the oil slick as a function of the time *t*.

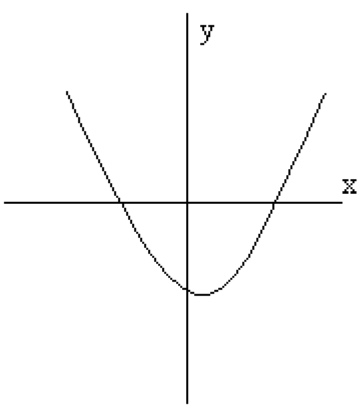
5.2: ONE-TO-ONE FUNCTIONS; INVERSE FUNCTIONS

When you are done with your homework, you should be able to…

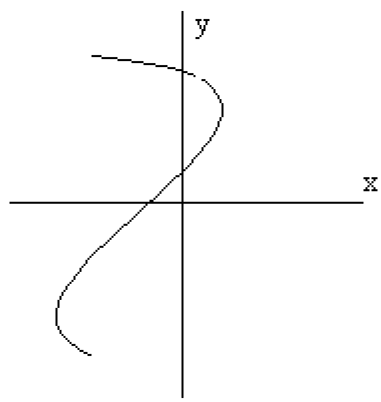
- π Determine Whether a Function is One-to-One
- π Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs
- π Obtain the Graph of the Inverse Function from the Graph of the Function
- π Find the Inverse of a Function Defined by an Equation

WARM-UP: Use the vertical line test to determine if the graphs of the relations are functions.

a.



b.



DETERMINE WHETHER A FUNCTION IS ONE-TO-ONE

A function f is one-to-one if no ______ in the _______________ is the _______________ of more than one ________ in the _________________. A function is not one-to-one if _________ different elements in the domain correspond to the ____________ element in the range.

Example 1: Determine whether the following functions are one-to-one.

a. For the following function, the domain represents the age of four males and the range represents the number of vehicles owned.

AGE	NUMBER OF VEHICLES
16	0
38	2
43	1
60	1

b. $\{(-6,1),(-1,3),(0,-1),(4,8)\}$

DEFINITION OF A ONE-TO-ONE FUNCTION

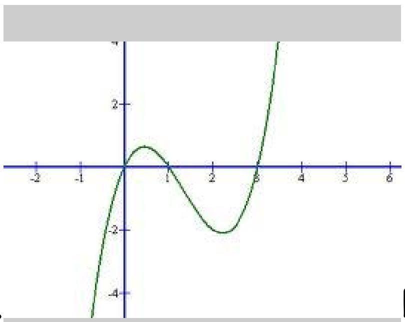
A function is one-to-one if and _____ ________________ inputs in the ___________________ correspond to _________ __________________ outputs in the _______________________. That is, if ___________ and ___________ are two different inputs of a function of f, then f is one-to-one if ___________________________.

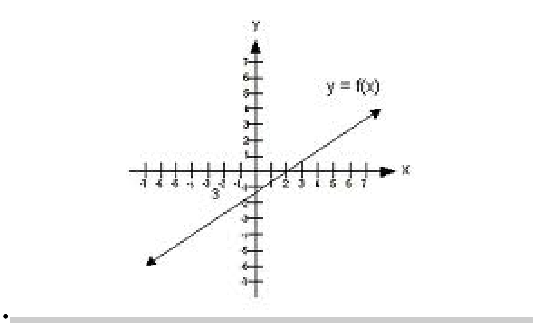
THEOREM: THE HORIZONTAL LINE TEST

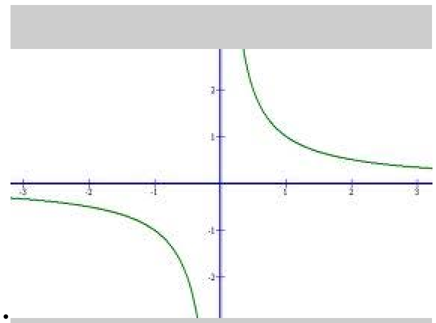
If every ______________________ line intersects the graph of f in

__________ __________________ one point, then f is one-to-one.

Example 2: Which of the following graphs represent one-to-one functions?

a. 

b. 

c. 

THEOREM

A function that is ______________________ on an interval I is a one-to-one function on I .

A function that is ______________________ on an interval I is a one-to-one function on I .

DEFINITION: INVERSE FUNCTION OF f

Suppose that f is a one-to-one function. Then, to each x in the domain of f , there is exactly ____________ ________ in the ______________ (because f is a __________________); and to each ______ in the ____________ of f , there is exactly __________ _______ in the domain (because f is __________________). The ______________________ from the _______________ of f back to the __________________ of f is called the __________________ function of f . The symbol ____________ is used to denote the _________________ of f . This symbol is read " f inverse".

Example 3: Find the inverse of each one-to-one function. State the domain and range of each inverse function.

a.

AGE	MONTHLY COST OF LIFE INSURANCE
30	$7.09
40	$8.40
45	$11.29

Domain: Range:

b. $\{(-6,1),(-1,3),(0,-1),(4,8)\}$

Domain: Range:

TWO FACTS ABOUT A ONE-TO-ONE FUNCTION f AND ITS INVERSE f^{-1}

1. ____________________ of ______ = ______________ of _______.

   ____________________ of ______ = ______________ of _______.

2. ______________________ = ____, where ____ is in the domain of ____.

   ______________________ = ____, where ____ is in the domain of ____.

Example 4: Show that each function is the inverse of the other.

$f(x) = 3x - 8$ and $g(x) = \frac{x+8}{3}$

GRAPHS OF A FUNCTION AND ITS INVERSE FUNCTION

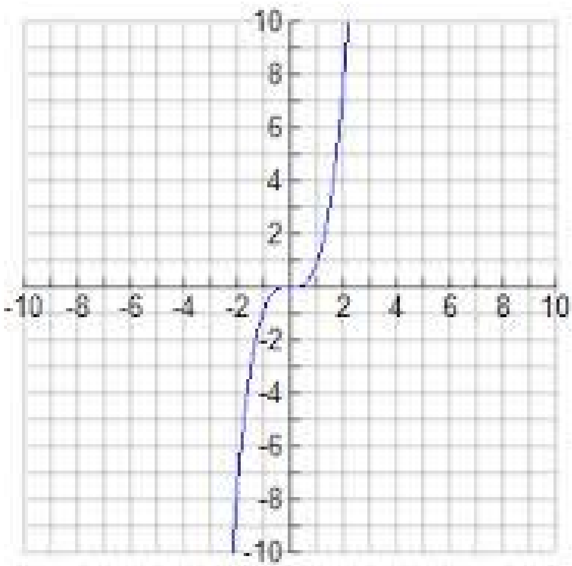
There is a ______________________ between the graph of a one-to-one function _____ and its inverse ________. Because inverse functions have ordered pairs with the coordinates __________________, if the point ___________ is on the graph of ______, the point ______________ is on the graph of __________. The points _______ and _______ are __________________ with respect to the line ________.

Therefore, the graph of ________ is a ____________________ of the graph of ______ about the line ___________.

THEOREM

The graph of a one-to-one function f and the graph of its ___________________ f^{-1} are ________________________ with respect to the line _________________.

Example 5: Use the graph of f below to draw the graph of its inverse function.



POINTS ON THE GRAPH OF f	POINTS ON THE GRAPH OF f^{-1}

STEPS FOR FINDING THE INVERSE OF A FUNCTION DEFINED BY AN EQUATION

The equation of the inverse of a function f can be found as follows:

1. Replace ________ with ______ in the equation for ________.

2. Interchange _____ and _____.

3. Solve for _____. If this equation does not define ____ as a function of ____, the function _____ does not have an ____________ function and this procedure ends. If this equation does define ____ as a function of ____, the function ____ has an inverse function.

4. If ____ has an inverse function, replace ____ in step 3 with ________. We can verify our result by showing that _________________ and _______________.

Example 6: Find an equation for $f^{-1}(x)$, the inverse function.

a. $f(x) = 4x$

b. $f(x) = \dfrac{2x-3}{x+1}$

APPLICATION

The function $T(g) = 1700 + 0.15(g - 17000)$ represent the 2011 federal income tax T (in dollars) due for a "married filing jointly" filer whose modified adjusted gross income is g dollars, where $17000 \le g \le 69000$.

a. What is the domain of the function T?

b. Given that the tax due T is an increasing linear function of modified adjusted gross income g, find the range of the function T.

c. Find adjusted gross income g as a function of federal income tax T. What are the domain and range of this function?

Section 5.3: EXPONENTIAL FUNCTIONS

When you are done with your homework you should be able to…

- π Evaluate Exponential Functions
- π Graph Exponential Functions
- π Define the Number *e*
- π Solve Exponential Equations

WARM-UP:

1. Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a+bi$.

   $\left(x^2-2\right)^2-\left(x^2-2\right)=6$

2. Use a calculator to evaluate

   a. $3^{2.2}$

   b. $3^{2.24}$

   c. $3^{2.236}$

   d. $3^{2.2361}$

   e. $3^{\sqrt{5}}$

LAWS OF EXPONENTS

If *s*, *t*, *a*, and *b* are real numbers with ____________ and ____________, then

EXPONENTIAL GROWTH

Suppose a function f has the following properties:

1. The value of f doubles with every 1-unit increase in the independent variable *x*, and
2. The value of f at $x = 0$ is 10, so ____________________.

x	$f(x)$
0	10
1	
2	
3	
4	

EXPONENTIAL FUNCTIONS

An exponential function is a function of the form

where ____ is a _____________ real number (___________), __________, and

______________ is a real number. The domain of f all real numbers, the base a is

the ______________ __________________, and because

________________________, we call C the _______________ ______________.

THEOREM

For an exponential function ________________________, where ____________

and _______________, if ____ is any real number, then

Example 1: Determine if the given function is an exponential function.

a. $f(x) = 3^x$

b. $g(x) = (-4)^{x+1}$

Example 2: Determine whether the given function is linear, exponential, or neither. For those that are linear, find a linear function that models the data. For those that are exponential, find an exponential function that models the data.

a.

x	$y = f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

b.

x	$y = f(x)$
-2	-1
-1	3
0	7
1	11
2	15

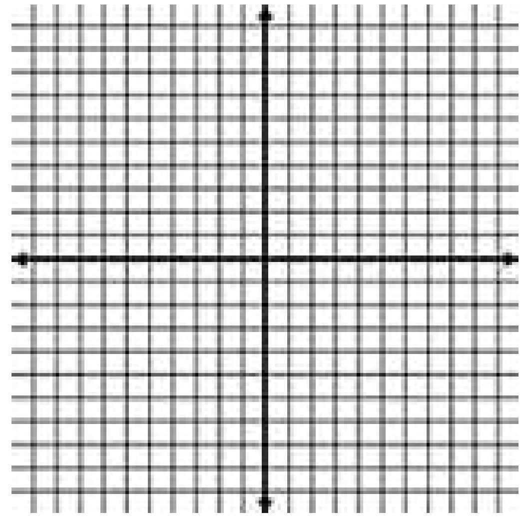
c.

x	$y = f(x)$
0	0
1	1
4	2
9	3
16	4

Example 3: Sketch the graph of each exponential function.

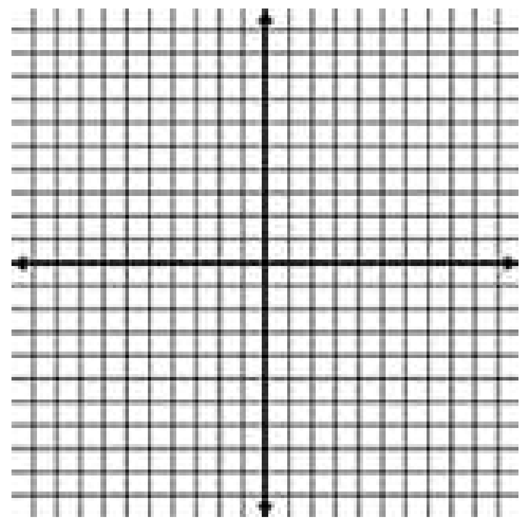
a. $f(x) = 2^x$

x	$f(x)$	$(x, f(x))$



b. $g(x) = 2^{-x}$

x	$g(x)$	$(x, g(x))$

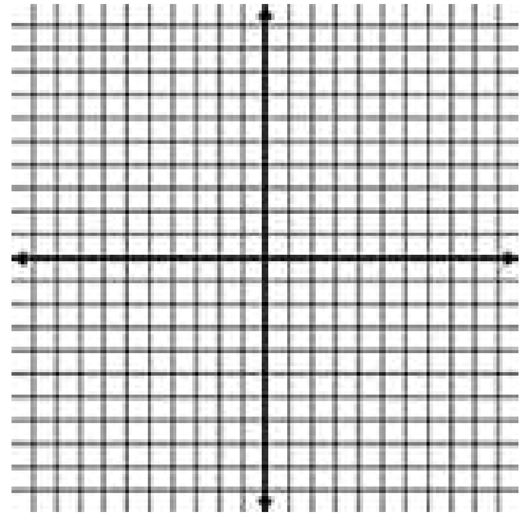


How are these two graphs related?

Example 4: Sketch the graph of each exponential function.

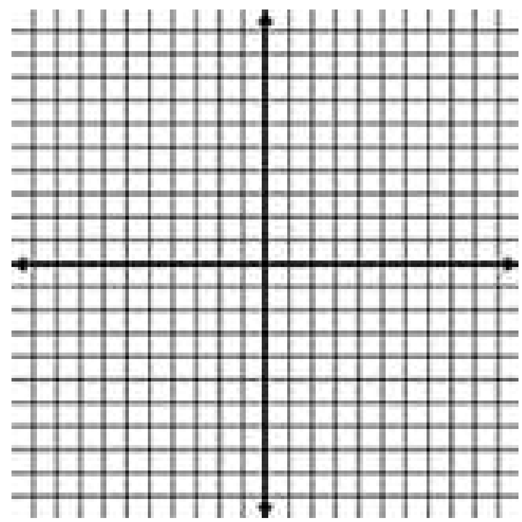
a. $f(x) = 3^x$

x	$f(x)$	$(x, f(x))$



b. $g(x) = 3^{x-1}$

x	$g(x)$	$(x, g(x))$



How are these two graphs related?

PROPERTIES OF THE EXPONENTIAL FUNCTIONS $f(x)=a^x,\ a>1$

1. The domain is the set of all real numbers or _______________ using interval notation. The range is the set of all _______________ real numbers or __________ using interval notation.
2. There are no _____________________; The *y*-intercept is ___________.
3. The ________________ is a _____________________ asymptote as ____________ [____________________].
4. ____________________, where ________________, is an ____________________ function and is ____________________________.
5. The graph of f contains the points ________________, _______________, and ____________________.
6. The graph of f is ____________________ and ________________________, with no _____________________ or _________________.

**Now look back at part a of each of the examples 3 and 4.

PROPERTIES OF THE EXPONENTIAL FUNCTIONS $f(x) = a^x,\ 0 < a < 1$

1. The domain is the set of all real numbers or _____________ using interval notation. The range is the set of all _____________ real numbers or ________ using interval notation.
2. There are no __________________; The *y*-intercept is __________.
3. The ______________ is a __________________ asymptote as __________ [_________________].
4. _________________, where ______________, is an _________________ function and is ________________________.
5. The graph of f contains the points ______________, _____________, and _________________.
6. The graph of f is _________________ and _____________________, with no __________________ or _______________.

**Now look back at example 3, part b.

n	$\left(1+\frac{1}{n}\right)^n$
1	
2	
5	
10	
100	

1000000000	

The irrational number _____, approximately _________________________, is called the ____________ base. The function ________________ is called the ________________ exponential function.

THE NUMBER *e*

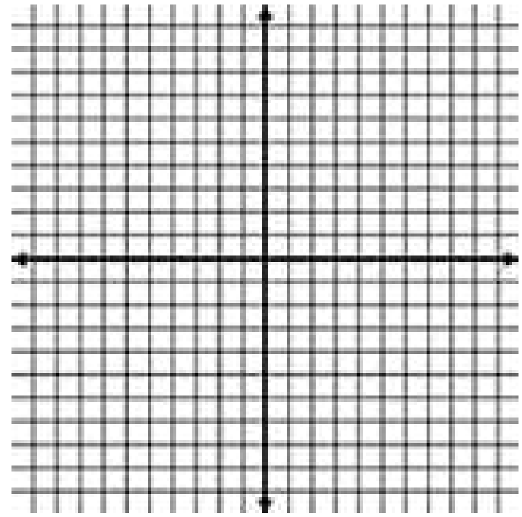
The number *e* is defined as the number that the expression

approaches as ___________________. In Calculus, this is expressed using _______________ notation as

Example 5: Sketch the graph of each exponential function.

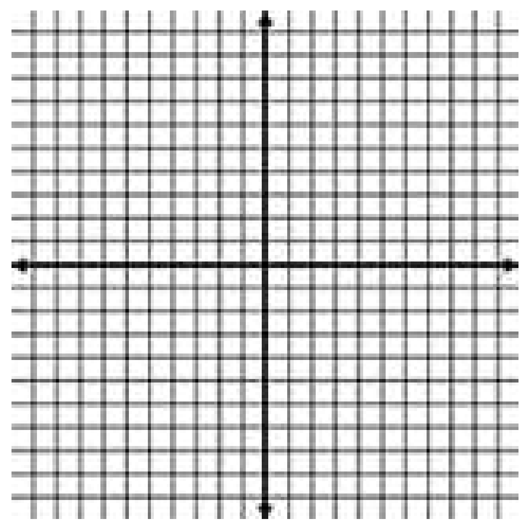
a. $f(x) = e^x$

x	$f(x)$	$(x, f(x))$



b. $g(x) = -e^x$

x	$g(x)$	$(x, g(x))$



How are these two graphs related?

EXPONENTIAL EQUATIONS

If ___________________________, then __________________________.

Example 6: Solve each equation. Verify your results using a graphing calculator.

a. $8^x = 8^{-2}$

b. $9^{-x+15} = 27^x$

c. $\left(e^{4}\right)^{x} \cdot e^{x^{2}} = e^{12}$

APPLICATIONS

1. The normal healing of wounds can be modeled by an exponential function. If A_0 represents the original area of the wound and A equals the area of the wound, then the function $A(n) = A_0 e^{-0.35n}$ describes the area of the wound after *n* days following an injury when no infection is present to retard the healing. Suppose that a wound initially had an area of 100 square millimeters.

   a. If healing is taking place, how large will the area of the wound be after 3 days?

   b. How large will it be after 10 days?

2. Suppose that a student has 500 vocabulary words to learn. If a student learns 15 words after 5 minutes, the function $L(t)=500\left(1-e^{-0.0061t}\right)$ approximates the number of words L that a student will learn after t minutes.

   a. How many words will the student learn after 30 minutes?

   b. How many words will the student learn after 60 minutes?

Section 5.4: LOGARITHMIC FUNCTIONS

When you are done with your homework you should be able to…

- π Change Exponential Statements to Logarithmic Statements
- π Evaluate Logarithmic Expressions
- π Determine the Domain of a Logarithmic Function
- π Graph Logarithmic Functions
- π Solve Logarithmic Equations

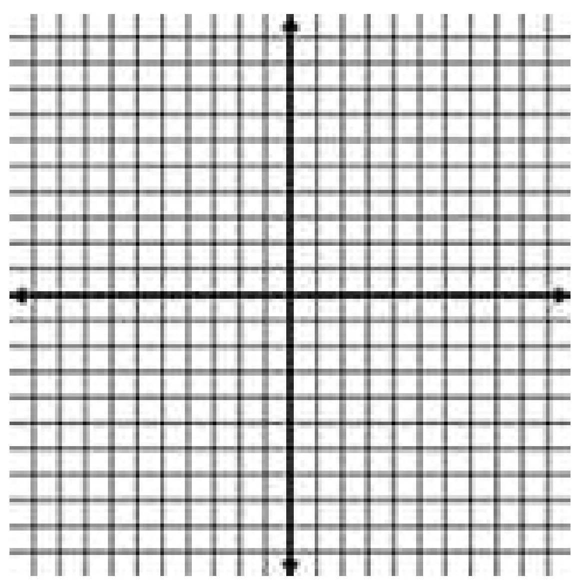
WARM-UP:

1. Solve.

$$\left(\frac{1}{64}\right)^{2x} = 16^{x^2-5}$$

2. Use the graph of $f(x) = 2^x$ to graph $f^{-1}(x)$.

POINTS ON THE GRAPH OF f	**POINTS ON THE GRAPH OF** f^{-1}



LOGARITHMIC FUNCTION

The logarithmic function to the base _____, where _________ and ________, is denoted by _____________________ (read as "_____ is the _____________ to the base _____ of _____") and is defined by

The domain of the logarithmic function _______________ is _______ or ____________ in interval notation.

INTERESTING FACTS…

__________ of the logarithmic function = ____________ of the ________________ function.

__________ of the logarithmic function = ____________ of the ________________ function.

PROPERTIES

__________________ (DEFINING EQUATION: _____________)

Domain: __________________ Range: _____________________

Example 1: Change each exponential statement to an equivalent statement involving a logarithm.

a. $16 = 4^2$

b. $e^{2.2} = M$

c. $3^x = 4.6$

Example 2: Change each logarithmic statement to an equivalent statement involving an exponent.

a. $\log_3\left(\frac{1}{9}\right) = -2$

b. $\log_6 2 = x$

c. $\log_e x = 5$

Example 3: When working this example, remember that the expression $\log_a x$ translates as "the power to which we raise *a* to get *x* is". Find the exact value of:

a. $\log_6\left(\frac{1}{216}\right)$

b. $\log_3 81$

c. $\log_e e$

Example 4: Find the domain of each logarithmic function.

a. $F(x) = \log_4(x+7)$

b. $h(x) = \log_e(x^2 - 16)$

c. $\log_7(-x)$

PROPERTIES OF THE LOGARITHMIC FUNCTION $f(x) = \log_a x$

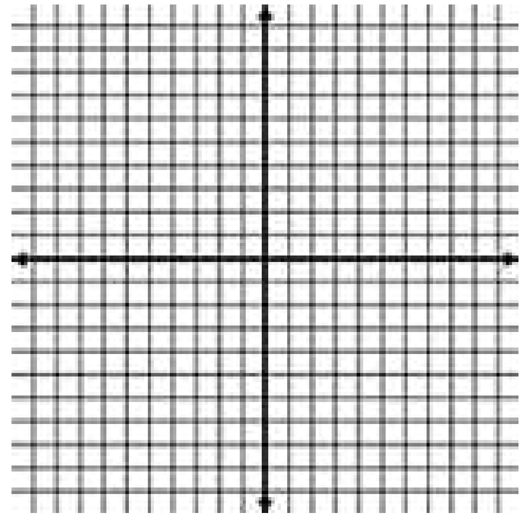
1. The domain is the set of __________________ real numbers or ___________ using interval notation. The range is the set of all _____________ numbers or _________ using interval notation.
2. The *x*-intercept of the graph is _______; there is no ___________________.
3. The ______________(_________) is a ___________________ asymptote of the graph.
4. A logarithmic function is __________________ if ____________, and ___________________ if ____________________.
5. The graph of f contains the points ______________, _____________, and __________________.
6. The graph of f is __________________ and _____________________, with no ___________________ or _______________.

**See Warm-up 2 to see the graph of $f(x) = \log_2 x$.

Example 5: Sketch the graph of the logarithmic function.

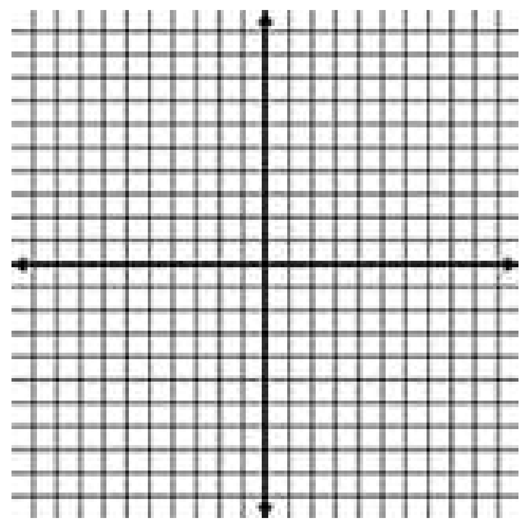
a. $f(x) = \log_3 x$

x	$f(x)$	$(x, f(x))$



b. $f(x) = \log_3(x-4)$

x	$g(x)$	$(x, g(x))$



How are these two graphs related?

If the base of a logarithmic function is ______, then we have the ________________ _________________ function. We often use this function in applications. The symbol _______ denotes the natural logarithmic function. This comes from the Latin phrase *logarithmus naturalis*.

NATURAL LOGARITHMIC FUNCTION

__________________________ if and only if ________________________

If the base of a logarithmic function is the number _______, then we have the ___________________ ____________________ function. If the base of the logarithmic function is not indicated, it is understood to be ________.

COMMON LOGARITHMIC FUNCTION

__________________________ if and only if ________________________

Example 6: Use a calculator to evaluate each expression. Round your answer to three decimal places.

a. $\frac{\ln 5}{8}$

b. $\frac{\log \frac{2}{3}}{-0.2}$

c. $\frac{\log 15 + \log 20}{\ln 15 + \ln 20}$

Example 7: Solve each equation. Verify your results using a graphing calculator.

a. $\log_5 x = 3$

b. $\log_3 (3x - 2) = 2$

c. $\ln e^{-2x} = 8$

d. $\log_6 36 = 5x + 3$

e. $\log x^2 = 4$

f. (Use your graphing calculator to solve this one☺)
$4e^{x+1} = 5$

APPLICATIONS

1. The normal healing of wounds can be modeled by an exponential function. If A_0 represents the original area of the wound and A equals the area of the wound, then the function $A(n) = A_0 e^{-0.35n}$ describes the area of the wound after *n* days following an injury when no infection is present to retard the healing. Suppose that a wound initially had an area of 100 square millimeters.

    a. If healing is taking place, after how many days will the wound be one-half its original size?

    b. How long before the wound is 10% of its original size?

2. Psychologists sometimes use the function $L(t) = A\left(1 - e^{-kt}\right)$ to measure the amount L learned at time t. The number A represents the amount to be learned, and the number k measures the rate of learning. Suppose that a student has an amount A of 200 vocabulary words to learn. A psychologist determines that the student learned 20 vocabulary words after 5 minutes.

   a. Determine the rate of learning k.

   b. Approximately how many words will the student have learned after 10 minutes?

   c. After 15 minutes?

   d. How long does it take for the student to learn 180 words?

Section 5.5: PROPERTIES OF LOGARITHMS

When you are done with your homework, you should be able to…

- π Work with the Properties of Logarithms
- π Write a Logarithmic Expression as a Sum or Difference
- π Evaluate a Logarithm Whose Base is Neither 10 Nor *e*
- π Graph a Logarithmic Function Whose Base is Neither 10 Nor *e*

WARM-UP:

1. Show that $\log_a 1 = 0$.

2. Show that $\log_a a = 1$.

IN SUMMARY…

PROPERTIES OF LOGARITHMS

Let ___ and ___ be positive real numbers with _________, and let ___ be any real number.

1.

2.

Example 1: Evaluate.

a. $\log_6 6$

b. $\log_{12} 12^4$

c. $\log_9 1$

d. $7^{\log_7 24}$

THE PRODUCT RULE

Let ___, ___, and ___ be positive real numbers with _________.

The logarithm of a product is the ________ of the _________________.

Example 2: Expand each logarithmic expression.

a. $\log_6(6x)$

b. $\ln(x \cdot x)$

THE QUOTIENT RULE

Let ___, ___, and ___ be positive real numbers with ________.

The logarithm of a quotient is the _____________ of the ________________.

Example 3: Expand each logarithmic expression.

a. $\log\frac{1}{x}$

b. $\log_4\frac{x}{2}$

THE POWER RULE

Let ___, ___, and ___ be positive real numbers with ________, and let ___ be any real number.

The logarithm of a power is the _____________ of the ________________ and the ______________.

Example 4: Expand each logarithmic expression.

a. $\log x^2$

b. $\log_5 \sqrt{x}$

PROPERTIES FOR EXPANDING LOGARITHMIC EXPRESSIONS

For_____ and ______:

1. ________________ $= \log_b M + \log_b N$
2. ________________ $= \log_b M - \log_b N$
3. ________________ $= p \log_b M$

Example 5: Expand each logarithmic expression.

a. $\log x^4 \sqrt[3]{y-1}$

b. $\log_2 \sqrt{\frac{x^2+5}{12y^6}}$

PROPERTIES FOR CONDENSING LOGARITHMIC EXPRESSIONS

For_____ and ______:

1. ____________________= $\log_b (MN)$

2. ____________________= $\log_b \frac{M}{N}$

3. ____________________= $\log_b M^p$

Example 6: Write as a single logarithm.

a. $3\ln x - \frac{1}{4}\ln(x-2)$

b. $\log_4 5 + 12\log_4 (x+y)$

For any logarithmic bases ___ and ___, and any positive number ___, _______:

If _______________, then _________________.

If _______________, then _________________.

THE CHANGE-OF-BASE PROPERTY

If _______, __________, and ___ are positive real numbers, then

Why would we use this property?

Example 7: Use common logarithms to evaluate $\log_5 23$.

Example 8: Use natural logarithms to evaluate $\log_5 23$.

What did you find out???

APPLICATION

1. If $f(x)=\log_a x$, show that the difference quotient $\frac{f(x+h)-f(x)}{h}=\log_a\left(1+\frac{h}{x}\right)^{1/h}, h\neq 0$.

Section 5.6: LOGARITHMIC AND EXPONENTIAL EQUATIONS

When you are done with your homework, you should be able to…

- π Solve Logarithmic Equations
- π Solve Exponential Equations
- π Solve Logarithmic and Exponential Equations Using a Graphing Calculator

WARM-UP:

Solve.

$$\frac{x^2 - x}{5} = \frac{2}{5}$$

SOLVING EXPONENTIAL EQUATIONS BY EXPRESSING EACH SIDE AS A POWER OF THE SAME BASE

If ______________________, then ___________________.

1. Rewrite the equation in the form ___________________.
2. Set _______________.
3. Solve for the variable.

Example 1: Solve.

a. $10^{x^2-1}=100$

b. $4^{x+1}=8^{3x}$

USING LOGARITHMS TO SOLVE EXPONENTIAL EQUATIONS

1. Isolate the ________________ expression.
2. Take the _________________ logarithm on both sides for base _____. Take the ________________ logarithm on both sides for bases other than 10.
3. Simplify using one of the following properties:
4. Solve for the variable.

Example 2: Solve.

a. $e^{2x}-6=32$

b. $\frac{3^{x-1}}{2}=5$

c. $10^{x}=120$

USING EXPONENTIAL FORM TO SOLVE LOGARITHMIC EQUATIONS
1. Express the equation in the form ________________. 2. Use the definition of a logarithm to rewrite the equation in exponential form: 3. Solve for the variable. 4. Check proposed solutions in the ______________ equation. Include in the solution set only values for which _________.

Example 3: Solve.

a. $\log_3 x - \log_3\left(x-2\right) = 4$

b. $\log x + \log\left(x+21\right) = 2$

USING THE ONE-TO-ONE PROPERTY OF LOGARITHMS TO SOLVE LOGARITHMIC EQUATIONS

1. Express the equation in the form __________________. This form involves a _______________ logarithm whose coefficient is ___ on each side of the equation.

2. Use the one-to-one property to rewrite the equation without logarithms:

3. Solve for the variable.
4. Check proposed solutions in the _______________ equation. Include in the

   solution set only values for which __________ and ____________.

Example 4: Solve.

a. $2\log_6 x - \log_6 64 = 0$

b. $\log(5x+1) = \log(2x+3) + \log 2$

Example 5: Use a graphing calculator to solve each equation. Express your answer rounded to two decimal places.

a. $e^{2x} = x + 2$

b. $\ln 2x = -x + 2$

c. $\log_2(x-1) - \log_6(x+2) = 2$

Example 6: Solve each equation. Express irrational solutions in exact form and as a decimal rounded to three decimal places.

a. $\log_2(3x+2)-\log_4 x=3$

b. $\dfrac{e^x+e^{-x}}{2}=3$

Section 5.7: FINANCIAL MODELS

When you are done with your homework, you should be able to…

- π Determine the Future Value of a Lump Sum of Money
- π Calculate Effective Rates of Return
- π Determine the Present Value of a Lump Sum of Money
- π Determine the Rate of Interest or Time Required to Double a Lump Sum of Money

WARM-UP: If you borrow $8,000, and, after 10 months, pay off the loan in the amount of $8,500, what per annum rate of interest was charged?

SIMPLE INTEREST FORMULA

If a principal of *P* dollars is borrowed for a period of *t* years at a per annum interest rate *r*, expressed as a ____________________, the interest *I* charged is

** *Simple Interest*

FORMULAS FOR COMPOUND INTEREST

After _____ years, the balance ____, in an account with principal ____ and annual interest rate ____ (in decimal form) is given by the following formulas:

1. For ___ compounding interest periods per year:

2. For continuous compounding:

**A is referred to as the _______________ value of the account and P is referred to as the ________________ value.

Example 1: Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% if the money is

a. compounded semiannually:

b. compounded monthly:

c. compounded continuously:

EFFECTIVE RATE OF INTEREST

The ____________ ___________ of __________________ _____ of an investment earning an annual interest rate ____ is given by

Compounding ____ times per _____:

________________ ________________:

Example 2: Find the principal needed now to get each amount; that is, find the present value.

a. To get \$800 after $3\frac{1}{2}$ years at 7% compounded monthly.

b. To get \$800 after $3\frac{1}{2}$ years at 7% compounded continuously.

Example 3: Find the effective rate of interest.

a. For 5% compounded quarterly.

b. For 5% compounded continuously.

Example 4: Determine the rate that represents the better deal.

9% compounded quarterly or 8.8% compounded daily.

APPLICATIONS

1. How many years will it take to for an initial investment of $25,000 to grow to $80,000? Assume a rate of interest of 7% compounded continuously.

2. Colleen and Bill have just purchased a home for $650,000, with the seller holding a second mortgage of $100,000. They promise to pay the seller $100,000 plus all accrued interest 5 years from now. The seller offers them three interest options on the second mortgage.
    a. Simple interest at 12% per annum
    b. $11\frac{1}{2}$% interest compounded monthly
    c. $11\frac{1}{4}$% interest compounded continuously

    Which option is best for Colleen and Bill?

Section 5.8: EXPONENTIAL GROWTH AND DECAY MODELS; NEWTON'S LAW; LOGISTIC GROWTH AND DECAY MODELS

When you are done with your homework, you should be able to…

- π Find Equations of Populations That Obey the Law of Uninhibited Growth
- π Find Equations of Populations That Obey the Law of Decay
- π Use Newton's Law of Cooling
- π Use Logistic Models

WARM-UP: Graph $A(t)=500e^{0.02t}$ and $A(t)=500e^{-0.02t}$ on your graphing calculator. How are these graphs related?

EXPONENTIAL GROWTH AND DECAY MODELS

The mathematical model for **exponential growth** or **decay** is given by

- If ___________, the function models the amount, or size, of a _______________ entity. ________ is the ________________ amount, or size, of the growing entity at time _____________, ________ is the amount at time _____, and ______ is a constant representing the ________________ rate.

- If ___________, the function models the amount, or size, of a _______________ entity. ________ is the ________________ amount, or size, of the decaying entity at time _____________, ________ is the amount at time _____, and ______ is a constant representing the ________________ rate.

Example 1: A culture of bacteria obeys the law of uninhibited growth.

a. If N is the number of bacteria in the culture and t is the time in hours, express N as a function of t.

b. If 500 bacteria are present initially, and there are 800 after 1 hour, how many will be present in the culture after 5 hours?

Example 2: The population of a Midwestern city follows the exponential law.

a. If N is the population of the city and t is time in years, express N as a function of t.

b. If the population doubled in size over an 18-month period and the current population is 20,000, what will the population be 2 years from now?

Example 3: A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

Example 4: A fossilized leaf contains 70% of its normal amount of carbon 14.

a. How old is the fossil? Use 5700 years as the half-life of carbon 14.

b. Using your graphing calculator, graph the relation between the percentage of carbon 14 remaining and time.

c. Using INTERSECT determine the time that elapses until half of the carbon 14 remains.

d. Verify the answer in part a.

NEWTON'S LAW OF COOLING

The temperature *u* of a heated object at a given time *t* can be modeled by the following function:

where *T* is the constant temperature of the surrounding medium, _____ is the initial temperature of the heated object, and _____ is a ________________ constant.

Example 5: A thermometer reading 72˚F is placed in a refrigerator where the temperature is a constant 38˚F.

a. If the thermometer reads 60˚F after 2 minutes, what will it read after 7 minutes?

b. How long will it take before the thermometer reads 39˚F?

c. Using your graphing calculator, graph the relation between temperature and time.

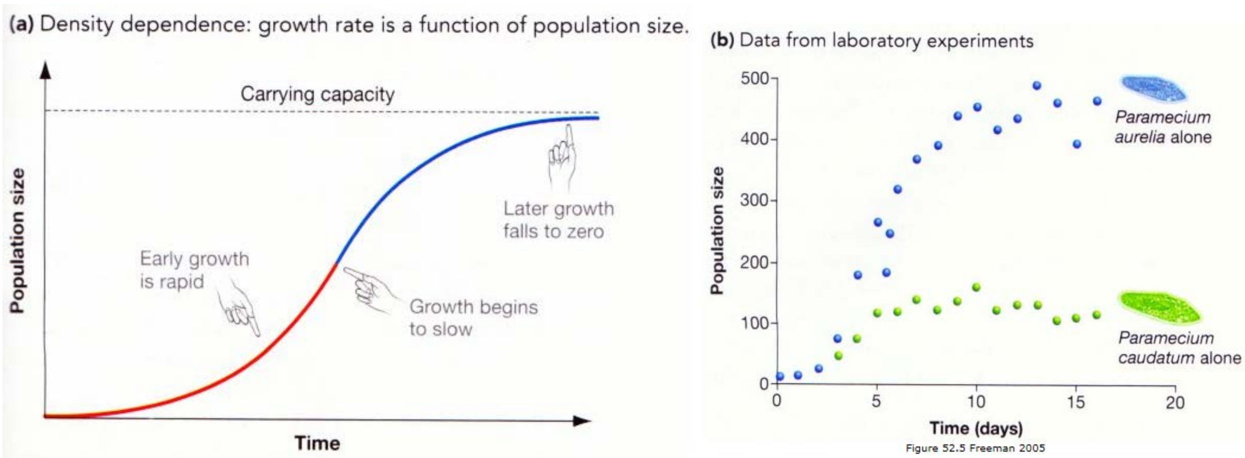
d. Using INTERSECT determine the time needed to elapse before the thermometer reads 45°F.

e. TRACE the function for large values of time. What do you notice about the temperature, *y*?

LOGISTIC MODEL

In a Logistic model, the population P after time t is given by function:

where a, b, and c are constants with ________ and ________. The model is a growth model if _____________; the model is a _____________ model if ___________.



PROPERTIES OF THE LOGISTIC MODEL

1. The domain is the set of all ___________ numbers. The range is the interval _____________, where c is the __________________ __________________.
2. There are no ___________________; the __________________ is ________.
3. There are ___________ ______________________ asymptotes: _____________ and ______________.
4. _____________ is an _________________ function if ________ and a ____________________ function if ___________.
5. There is an ____________________ point where __________ equals ______ of the carrying capacity. The inflection point is the point on the graph where the graph changes from being curved _______________ to _______________ for _______________ functions and the point where the graph changes from being curved _______________ to _______________ for _________________ functions.

Example 6: Often environmentalists capture an endangered species and transport the species to a controlled environment where the species can produce offspring and regenerate its population. Suppose that six American bald eagles are captured, transported to Montana, and set free. Based on experience, the environmentalists expect the population to grow according to the model $P(t)=\frac{500}{1+82.33e^{-0.162t}}$, where t is measured in years.

a. Determine the carrying capacity of the environment.

b. What is the growth rate of the bald eagle?

c. Use a graphing calculator to graph $P=P(t)$.

d. What is the population after 3 years?

e. When will the population be 300 bald eagles?

f. How long does it take the population to reach ½ of the carrying capacity?

Section 11.1: SYSTEMS OF LINEAR EQUATIONS; SUBSTITUTION AND ELIMINATION

When you are done with your homework you should be able to…

- π Solve Systems of Linear Equations by Substitution
- π Solve Systems of Linear Equations by Elimination
- π Identify Inconsistent Systems of Equations Containing Two Variables
- π Express the Solution of a System of Dependent Equations Containing Two Variables
- π Solve Systems of Three Equations Containing Three Variables
- π Identify Inconsistent Systems of Equations Containing Three Variables
- π Express the Solution of a System of Dependent Equations Containing Three Variables

WARM-UP:

Graph $5x+3y=21$ and $-x+2y=0$ using your graphing calculator.

What is the point of intersection?

What math problem have you solved?

SYSTEMS OF LINEAR EQUATIONS AND THEIR SOLUTIONS

We have seen that all _________________ in the form ________________ are straight _____________ when graphed. ___________ such equations are called a _________________ of ________________ __________________ or a _________________ __________________. A _________________ to a system of two ________________ equations in two _______________ is an _________________ _____________ that _______________ ________________ equations in the ________________.

Example 1: Determine whether the given ordered pair is a solution of the system.

a.

$$(-2,-5)$$
$$6x-2y=-2$$
$$3x+\ \ y=-11$$

b.

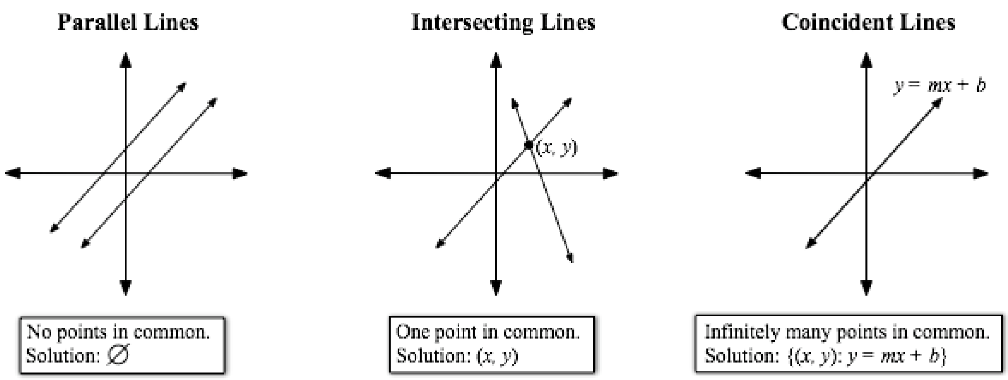
$$(10,7)$$
$$6x-\ \ 5y=25$$
$$4x+15y=13$$

SOLVING LINEAR SYSTEMS BY GRAPHING

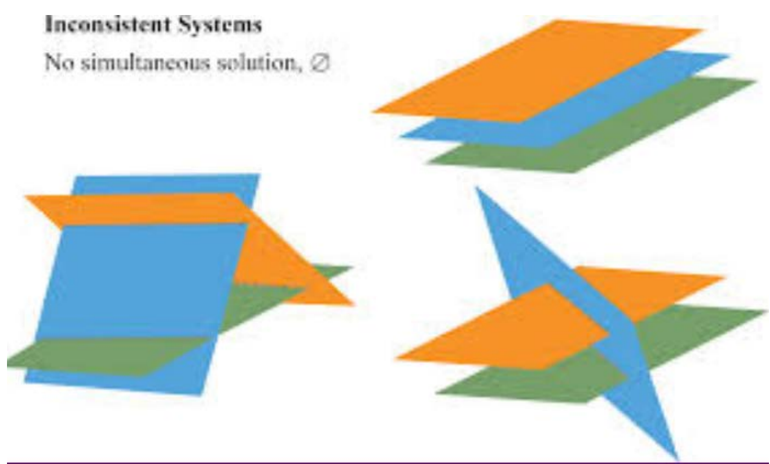
The ______________ of a _________________ of linear equations consists of values for the _____________________ that are ___________________ of each _____________________ in the _____________________. To ______________ a system means to find _____________ solutions of the ___________________.

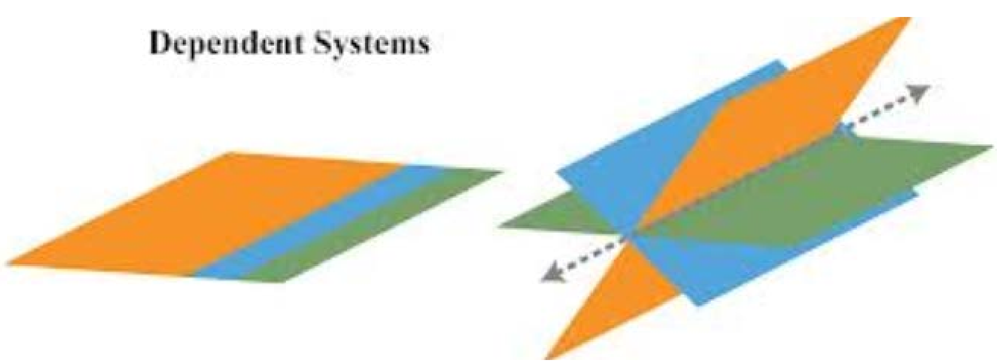
TYPES OF SOLUTIONS

2 Equations, 2 Variables

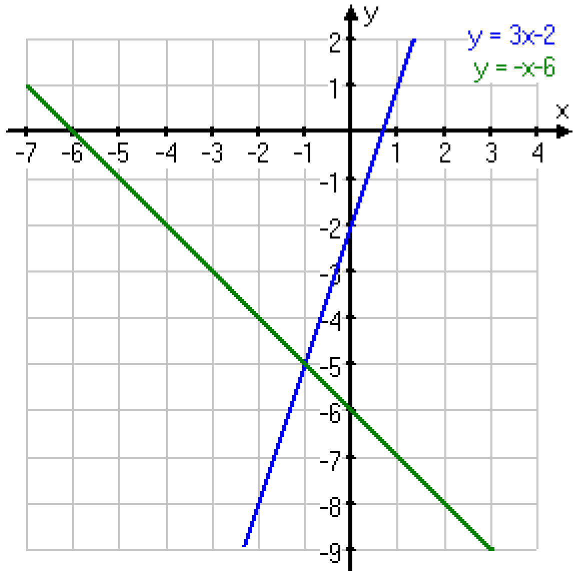


3 Equations, 3 Variables





Example 2: Use the graph below to find the solution of the system of linear equations.



RULES FOR OBTAINING AN EQUIVALENT SYSTEM OF EQUATIONS

1. ____________________ any two equations of the system.
2. ____________________ or ___________________ each side of an equation by the same ___________________ constant.
3. _____________________ any equation in the system by the ____________ or ___________________ of that equation and a __________________ multiple of any other equation in the system.

Example 3: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

a.

$$\begin{aligned} 5x + 2y &= -5 \\ 3x - y &= -14 \end{aligned}$$

b.

$$\begin{aligned} y &= 5x - 3 \\ y &= 2x - \frac{21}{5} \end{aligned}$$

c.

$$-x+3y=4$$
$$2x-6y=-8$$

Example 3: Solve the following systems of linear equations by the elimination method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$x+y=6$$
$$x-y=-2$$

b.

$$3x - y = 11$$
$$2x + 5y = 13$$

c.

$$4x - 2y = 2$$
$$2x - y = 1$$

Example 4: Solve each system of equations.

a.

$$\begin{cases} 2x + y = -4 \\ -2y + 4z = 0 \\ 3x - 2z = -11 \end{cases}$$

b.

$$\begin{cases} x - y + z = -4 \\ 2x - 3y + 4z = -15 \\ 5x + y - 2z = 12 \end{cases}$$

APPLICATIONS

1. The length of a fence required to enclose a rectangular field is 3000 meters. What are the dimensions of the field if it is known that the difference between its length and width is 50 meters?

2. A movie theater charges $9 for adults and $7 for students. On a day when 325 people paid an admission, the total receipts were $2495. How many who paid were adults? How many were students?

3. Kelly has $20000 to invest. As her financial planner, you recommend that she diversify into three investments: Treasury bills that yield 5% simple interest, Treasury bonds that yield 7% simple interest, and corporate bonds that yield 10% simple interest. Kelly wishes to earn $1390 per year in income. Also, Kelly wants her investment in Treasury bills to be $3000 more than her investment in corporate bonds. How much money should Kelly place in each investment?

4. The average airspeed of a single-engine aircraft is 150 mph. If the aircraft flew the same distance in 2 hours with the wind as it flew in 3 hours against the wind, what was the wind speed?

5. Find real numbers *a*, *b*, and *c* so that the graph of the function $y = ax^2 + bx + c$ contains the points $(-1,-2)$, $(1,-4)$, and $(2,4)$.

11.5: PARTIAL FRACTION DECOMPOSITION

When you are done with your homework, you should be able to…

- π Decompose $\frac{P}{Q}$, where Q Has Only Nonrepeated Linear Factors
- π Decompose $\frac{P}{Q}$, where Q Has Repeated Linear Factors
- π Decompose $\frac{P}{Q}$, where Q Has a Nonrepeated Irreducible Quadratic Factor
- π Decompose $\frac{P}{Q}$, where Q Has a Repeated Irreducible Quadratic Factor

WARM-UP:

Add $\frac{3}{x(x-1)^2}$ and $\frac{5}{x-1}$.

(CASE 1) Q HAS ONLY NONREAPEATED LINEAR FACTORS

Under the assumption that Q has only ___________________ linear factors, the polynomial Q has the form

where no two of the numbers ________________________ are equal. In this case, the partial fraction decomposition of ______________ is of the form

where the numbers _______________________ are to be determined.

Example 1: Write the partial fraction decomposition of each rational expression.

a.

$$\frac{3x}{(x+2)(x-1)}$$

b.

$$\frac{x^2 - x - 8}{(x+1)(x^2 + 5x + 6)}$$

(CASE 2) Q HAS REAPEATED LINEAR FACTORS

If the polynomial Q has a ___________________ linear factor, say

____________, ______________, *n* is an ________________, then, in the

partial fraction decomposition of ____________, we allow for the terms

where the numbers ______________________ are to be determined.

Example 2: Write the partial fraction decomposition of each rational expression.

$$\frac{x+1}{x^2(x-2)}$$

(CASE 3) Q CONTAINS A NONREAPEATED IRREDUCIBLE QUADRATIC FACTOR

If Q contains a ____________________ irreducible quadratic factor of the form

_________________________, then, in the partial fraction decomposition of

_____________, allow for the term

where the numbers _______________________ are to be determined.

Example 3: Write the partial fraction decomposition of each rational expression.

$$\frac{1}{\left(x^2+4\right)\left(x+1\right)}$$

(CASE 4) Q CONTAINS A REAPEATED IRREDUCIBLE QUADRATIC FACTOR

If the polynomial Q contains a ___________________ irreducible quadratic factor of the form ________________________, ____________, *n* is an _______________, then, in the partial fraction decomposition of _________, allow for the terms

where the numbers ______________________________ are to be determined.

Example 4: Write the partial fraction decomposition of each rational expression.

a.

$$\frac{x^3+1}{\left(x^2+16\right)^2}$$

b.

$$\frac{x^2+1}{x^3+x^2-5x+3}$$